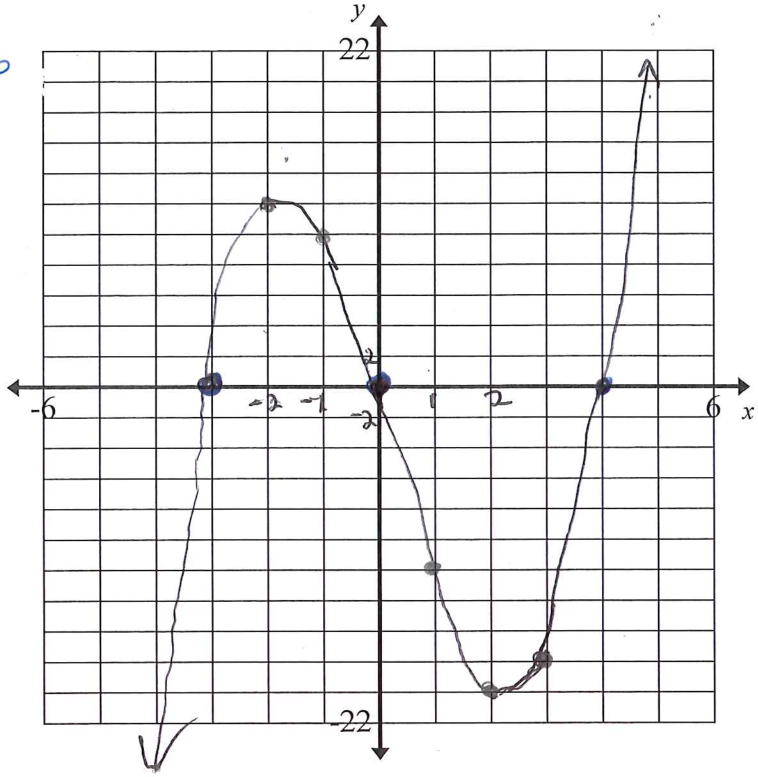
Name ______________________________ Period __________

6.1A Graphing Cubic Functions: Significant Features

1. Enter the equation $y = x^3 - x^2 - 12x$ into a graphing calculator and use a table of values to draw the graph.

a) Over what interval(s) is the graph increasing?

$x < -1.69$ & $x > 2.36$
(graphing calc)



b) Over what interval(s) is the graph decreasing?

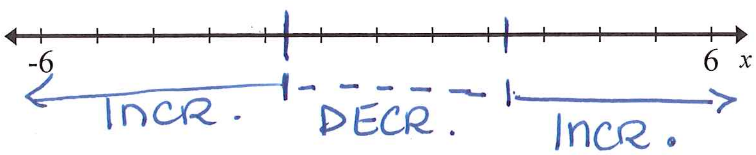
$-1.69 < x < 2.36$
(graphing calc)

c) Identify any maximum or minimum values, recording what are they and where they occur?

rel max $(-1.69, 12.56)$
rel min $(2.36, -20.75)$
These occur at turning points.

d) Identify the x-intercept(s).

$(-3, 0)$
$(0, 0)$
$(4, 0)$



e) Identify the y-intercept(s).

$(0, 0)$

f) Are there any restrictions on the domain and range? If yes, what are they?

No

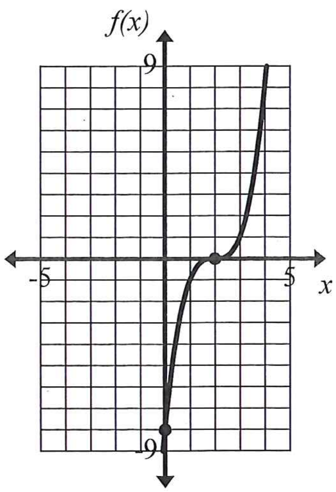
Name ______________________________ Period __________

6.1A *Graphing Cubic Functions: Significant Features*

2. A *cubic function* is described by $f(x) = ax^3 + bx^2 + cx + d$, where *a*, *b*, *c*, and *d* are the coefficients and $a \neq 0$. Graphs of cubic functions show a bit more variety than those for linear or quadratic functions. Here are some examples of cubic functions with their graphs:

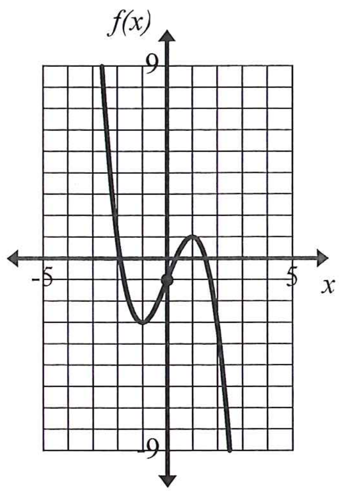
[A]

$f(x) = x^3 - 6x^2 + 12x - 8$



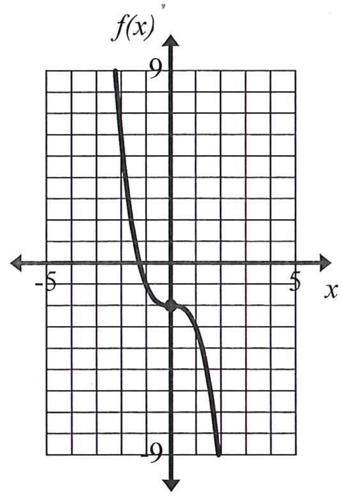
[B]

$f(x) = -x^3 + 3x - 1$



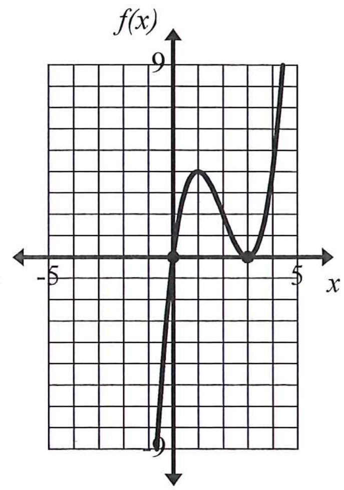
[C]

$f(x) = -x^3 - 2$



[D]

$f(x) = x^3 - 6x^2 + 9x$



a) Identify the lead coefficient of each cubic function.

Function **[A]**: 1

Function **[B]**: −1

Function **[C]**: −1

Function **[D]**: 1

b) Make a conjecture about what influence the **sign** of the lead coefficient, *a*, has on the shape of the graph.

If $a > 0$, the branches that tend to infinity both have a positive slope. ↙↗

If $a < 0$, " " " " " " " " negative slope. ↖↘

c) How would you describe the different types of cubic graphs? What do they have in common? How are they different?

They look like elongated "S" or "Z" curves rotated 90°

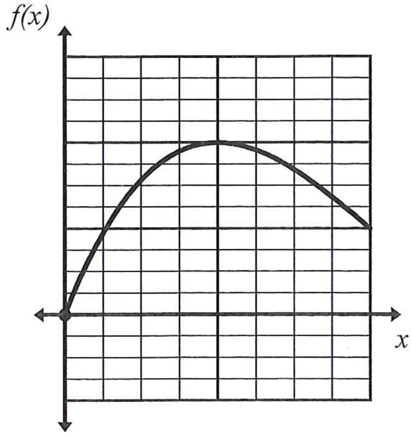
Common feature: The ends of the S or Z curve tend to $\pm\infty$.

Different feature: The # of times the graph crosses the x-axis, either 1, 2, or 3 times.

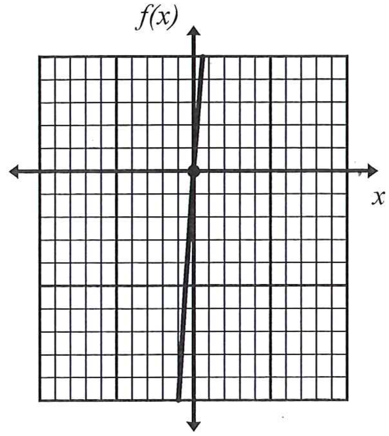
Name ____________________ Period __________

6.1A *Graphing Cubic Functions: Significant Features*

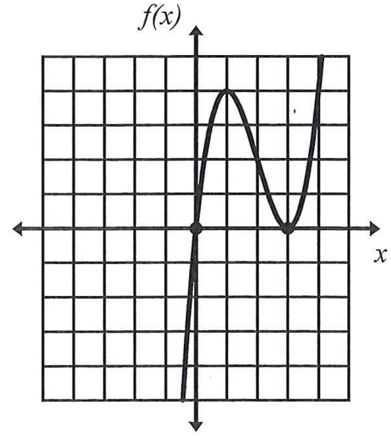
3. Kerry, Johanna, and Meng all used their graphing calculators to graph the function $f(x) = x^3 - 6x^2 + 9x$. They all entered the correct equation.



Kerry



Johanna



Meng

a) Explain why they got such different graphs for the same function.

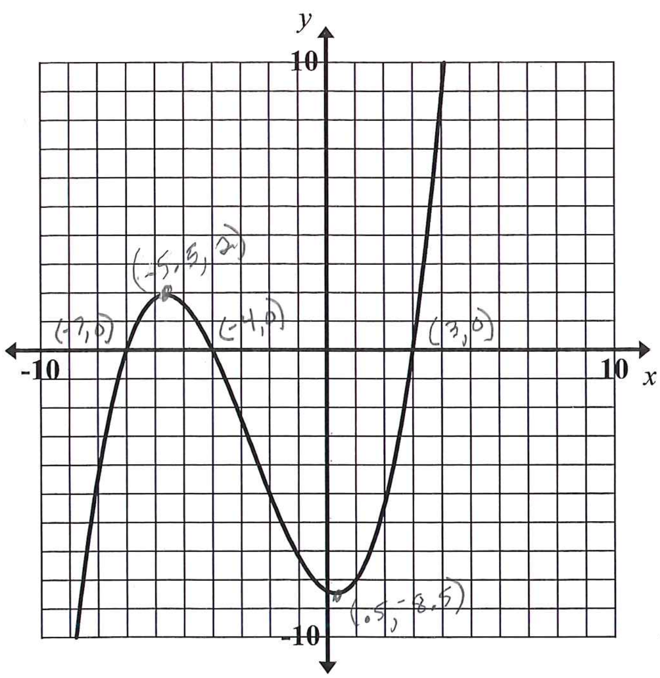
Different windows for x min, x max, y min, y max

b) Whose graph is the best representation of the cubic function? Why?

Meng, since you can see all the zeros, max & min

#4 – 6: For each graph, identify the significant features of the graph.

4.



Sign of the Lead Coefficient: Positive

Domain: all reals

Range: all reals

relative minimum: (0.5, -8.5)

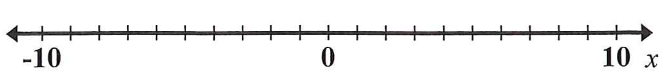
relative maximum: (-5.5, 2)

interval(s) where functions values are increasing: $X < -5.5$ and $X > 0.5$

interval(s) where functions values are decreasing: $-5.5 < X < 0.5$

x–intercept(s): (-7, 0), (-4, 0), (3, 0)

y-intercept: ≈ $(0, -8.\overline{3})$

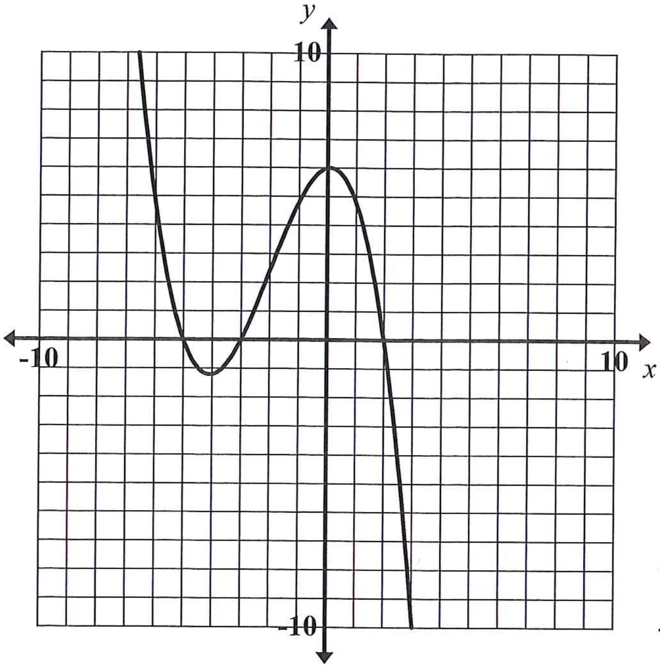


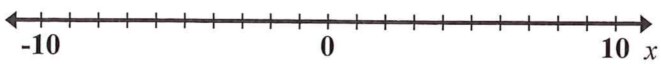
Name ____________________ Period ________

6.1A *Graphing Cubic Functions: Significant Features*

#4 – 6 (continued): For each graph, identify the significant features of the graph.

5.





Sign of the Lead Coefficient: negative

Domain: all reals

Range: all reals

relative minimum: (−4, −1.2*) approx. yvalue

relative maximum: (0, 6)

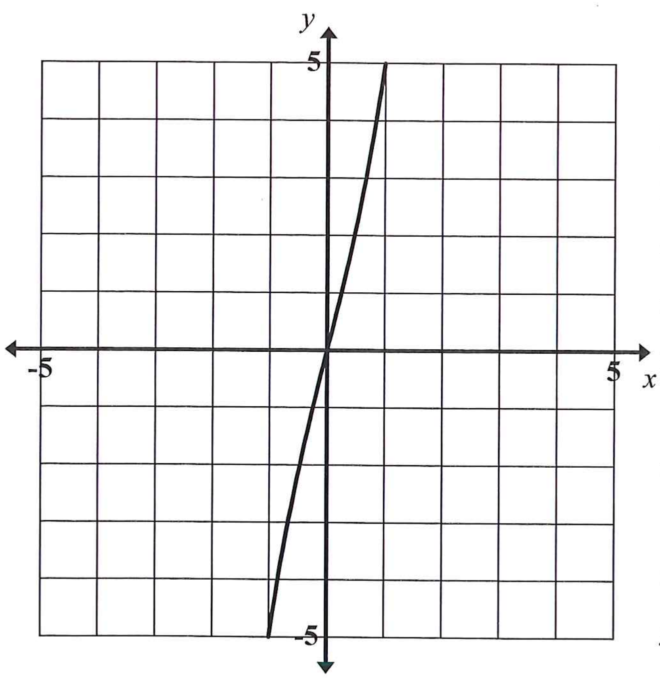
interval(s) where functions values are increasing: $-4 < x < 0$

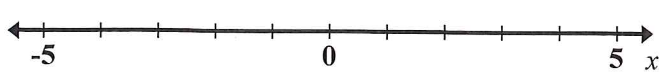
interval(s) where functions values are decreasing: $x < -4$ and $x > 0$

x–intercept(s): (−5,0), (−3,0), (2,0)

y-intercept: (0,6)

6.





Sign of the Lead Coefficient: positive

Domain: all reals

Range: all reals

relative minimum: none

relative maximum: none

interval(s) where functions values are increasing: $-\infty < x < \infty$

interval(s) where functions values are decreasing: none

x–intercept(s): (0,0)

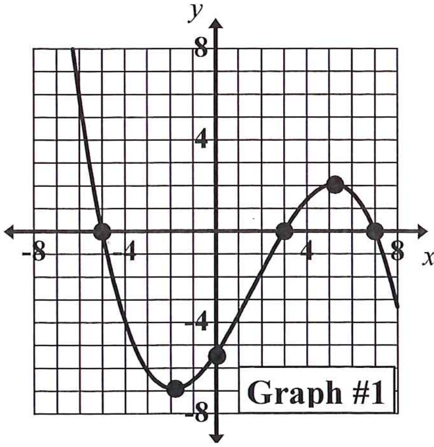
y-intercept: (0,0)

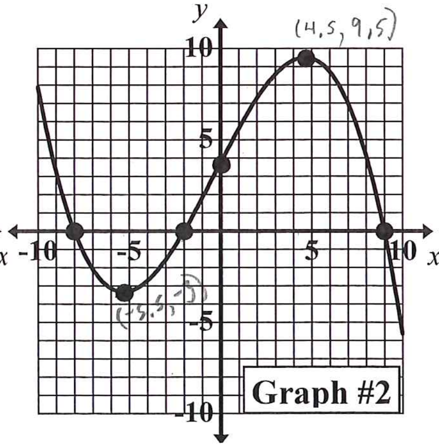
Name ______________________________ Period __________

6.1A *Graphing Cubic Functions: Significant Features*

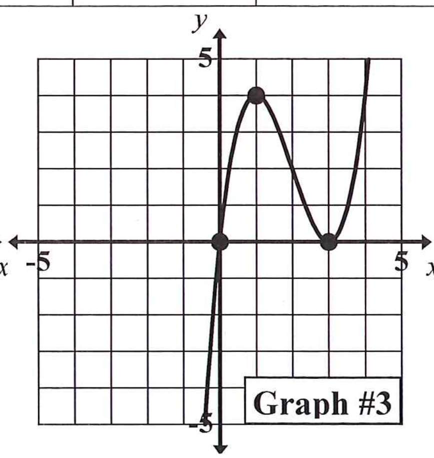
7. Determine which graph below has the identified value as a significant feature. Then use the graph to complete the table.

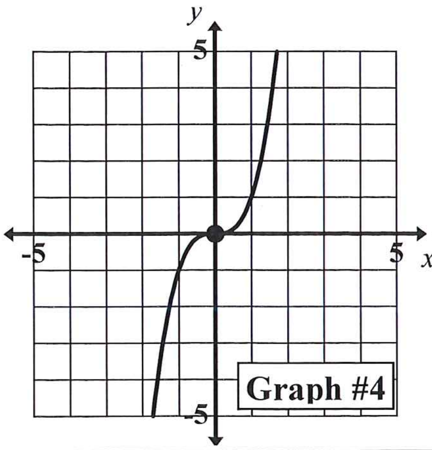
Graph #:	1	3	5	4	6	2
x-intercept(s):	(−5, 0) (3, 0) (7, 0)	(0, 0) (3, 0)	(1, 0)	(0, 0)	(−2.5, 0)	(−8, 0) (−2, 0) (9, 0)
y-intercept(s):	(0, −5.5)	(0, 0)	(0, 1)	(0, 0)	(0, 4)	(0, 4)
Relative Maximum:	(5, 2)	(1, 4)	none	none	(−1, 7)	(4.5, 9.5)
Relative Minimum:	(−2, −7)	(3, 0)	none	none	(1.1, 0.9)	(−5.5, −3.5)
interval(s) where function values are increasing:	$-2<x<5$	$x<1$, $x>3$	none	Increasing over entire domain	$x<-1$, $x>1.1$	$-5.5 < x < 4.5$
interval(s) where function values are decreasing:	$x<-2$, $x>5$	$1<x<3$	Decreasing over entire domain	none	$-1<x<1.1$	$x<-5.5$, $x>4.5$

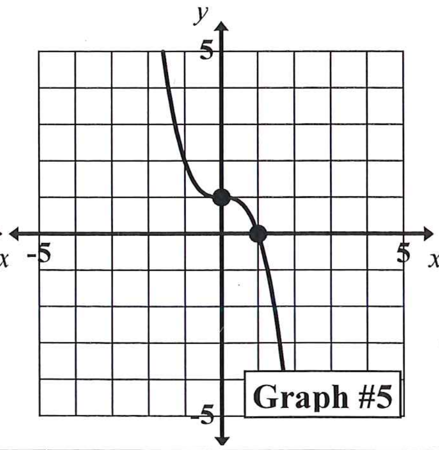

Graph #1

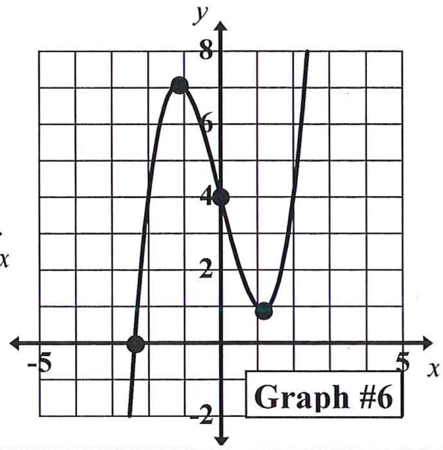


Graph #2


Graph #3


Graph #4


Graph #5


Graph #6

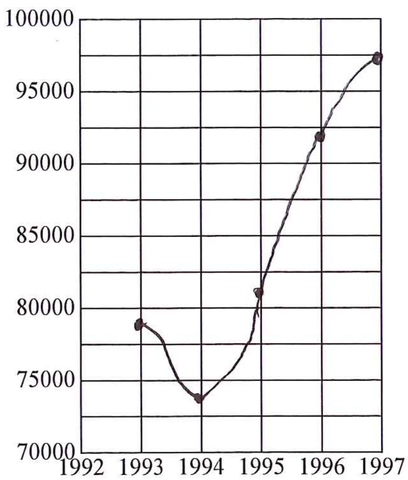
Name ______________________ Period __________

6.1A *Graphing Cubic Functions: Significant Features*

8. Draw a graph for the sets of data as reported in the USDA Statistical Highlights. From the graph, determine whether the data could be modeled with a cubic function. Give reasons for your answers.

a) Number of acres of fresh carrots harvested in the United States.

Year	Acres
1993	78,220
1994	74,630
1995	81,120
1996	92,160
1997	97,460

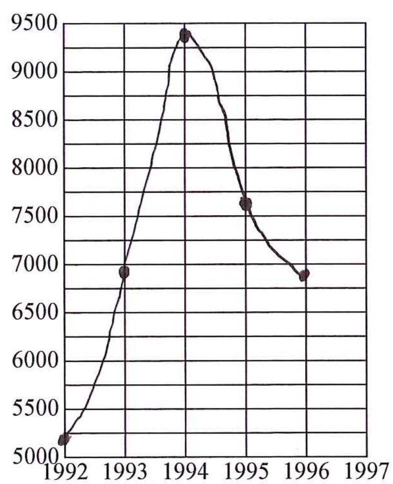


Possible cubic function? yes

If before year 1993, the # acres was < 74,630 and declining each previous year, then the end beh would be ↙↗.

b) Number of bales of cotton (in thousands) exported.

Year	Acres
1992	5,200
1993	6,860
1994	9,400
1995	7,680
1996	6,870

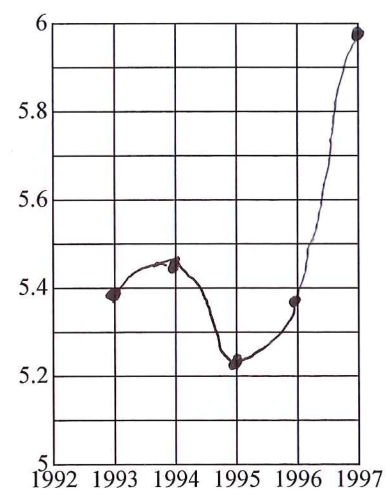


Possible cubic function? yes

If sometime after 1996 the # bales was > 9400 thousand and increasing each succeeding year, the end beh would then be ↙↗.

c) Yield per acre (in thousands) for processed cucumbers.

Year	Acres
1993	5.38
1994	5.44
1995	5.22
1996	5.37
1997	5.98



Possible cubic function? yes

If before the year 1993, the yield/acre was < 5.22 thousand. and decreasing each previous year, the end beh would then be ↙↗.

9. Write a cubic function that has the following end behavior: (↙, ↗)

one ex: $f(x) = x^3 + x + 1$

10. Write a cubic function that has the following end behavior: (↖, ↘)

one ex: $g(x) = -2x^3 + 3x^2 - x + 4$

Section 6.1A

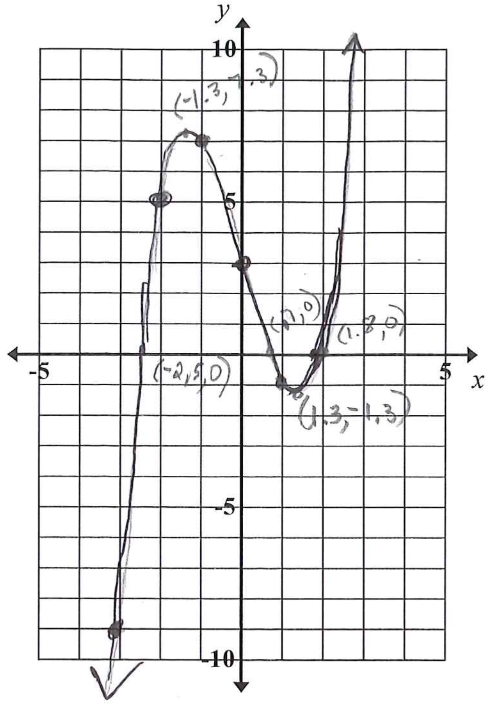
Name ______________________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#1 – 2: Use a table of values to graph each equation and identify the significant features of the graph.

1. $y = x^3 - 5x + 3$

x	y
-3	-9
-2	5
-1	7
0	3
1	-1
2	1
3	15



Sign of the Lead Coefficient : Positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: (1.3, -1.3)

Relative maximum: (-1.3, 7.3)

Interval(s) where function values are increasing: $x < -1.3$, $x > 1.3$

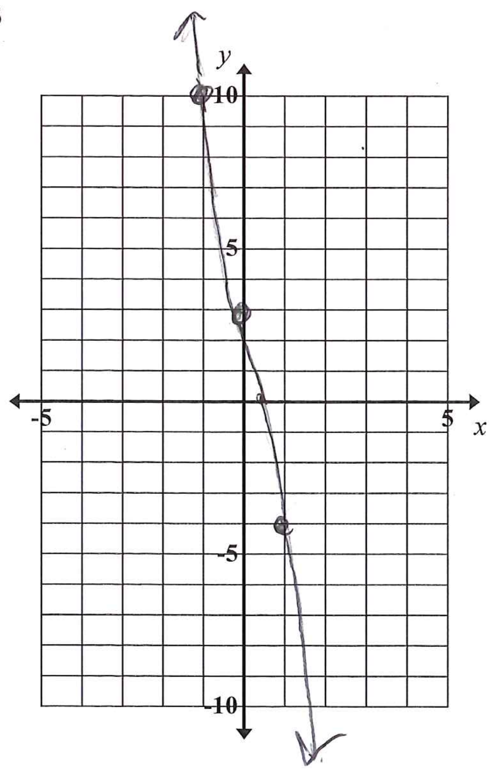
Interval(s) where function values are decreasing: $-1.3 < x < 1.3$

x-intercepts(s): (-2.5, 0), (0.7, 0), (1.8, 0)

y-intercept(s): (0, 3)

2. $y = -2x^3 - 5x + 3$

x	y
-2	29
-1	10
0	3
1	-4
2	-23



Sign of the Lead Coefficient: Negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: none

Interval(s) where function values are decreasing: $-\infty < x < \infty$

x-intercepts(s): (0.54, 0)

y-intercept(s): (0, 3)

3. How does a cubic function in standard form $y = ax^3 + bx^2 + cx + d$, relate to the significant features of the graph? a determines the end behavior; if a > 0, end beh is ↙↗; if a < 0, end beh ↖↘.
d is the y-intercept

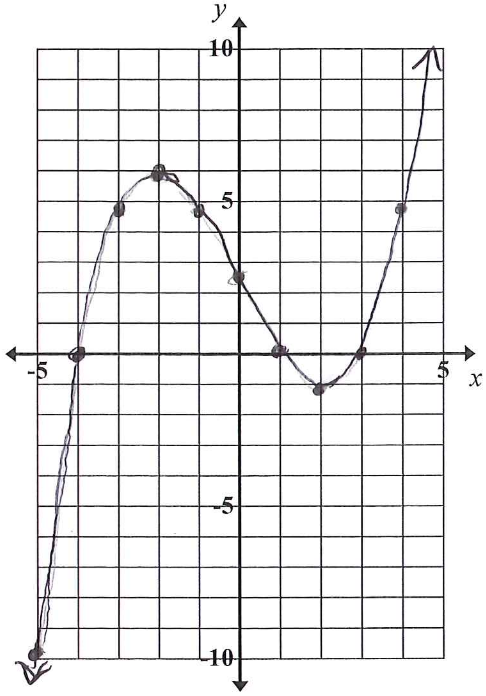
Name ______________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#4 – 5: Use a table of values to graph each equation and identify the significant features of the graph.

4. $y = 0.2(x+4)(x-3)(x-1)$

x	y
-5	-9.6
-4	0
-3	4.8
-2	6
-1	4.8
0	2.4
1	0
2	-1.2
3	0
4	4.8



Sign of the Lead Coefficient: positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: (2, -1.2)

Relative maximum: (-2, 6)

Interval(s) where function values are increasing: x < -2, x > 2

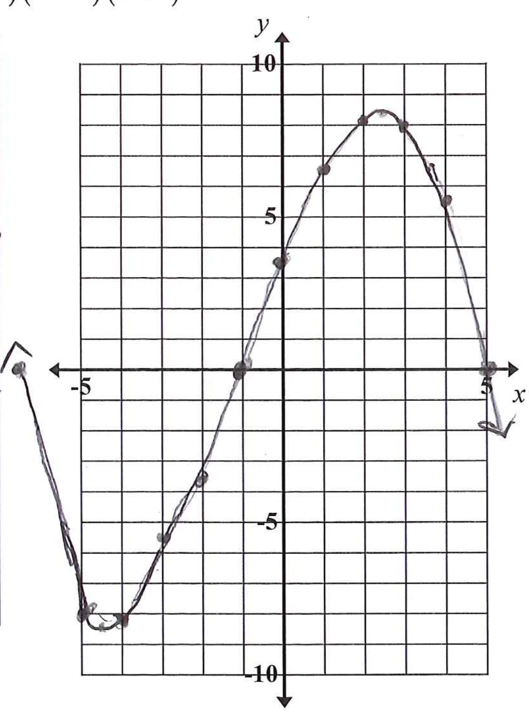
Interval(s) where function values are decreasing: -2 < x < 2

x-intercepts(s): (-4, 0), (1, 0), (3, 0)

y-intercept(s): (0, 2.4)

5. $y = -0.1(x-5)(x+7)(x+1)$

x	y
-5	-8
-4	-8.1
-3	-6.4
-2	-3.5
-1	0
0	3.5
1	6.4
2	8.1
3	8
4	5.5
5	0



Sign of the Lead Coefficient: negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: (-4.5, -8.3)

Relative maximum: (2.5, 8.3)

Interval(s) where function values are increasing: -4.5 < x 2.5

Interval(s) where function values are decreasing: x < -4.5, x > 2.5

x-intercepts(s): (-7, 0), (-1, 0), (5, 0)

y-intercept(s): (0, 3.5)

6. How does a cubic function in factored form $y = a(x-m)(x-n)(x-p)$, relate to the significant features of the graph? a determines the end behavior; if a > 0, ↙↗; if a < 0, ↖↘

m, n, p are the x-intercepts

a(-m)(-n)(-p) = y intercept

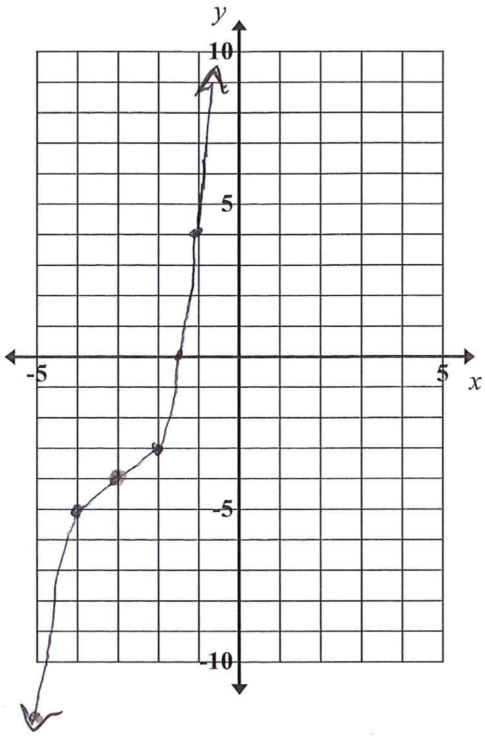
Name ______________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#7 – 8: Use a table of values to graph each equation and identify the significant features of the graph.

7. $y=(x+3)^3-4$

x	y
-5	-12
-4	-5
-3	-4
-2	-3
-1	4
0	23
1	60
2	121



Sign of the Lead Coefficient: positive

End behavior: ↙↗

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: $-\infty < x < \infty$

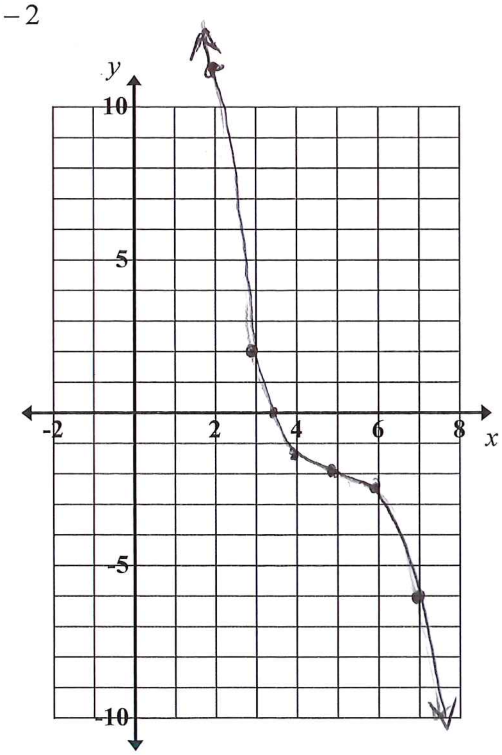
Interval(s) where function values are decreasing: none

x-intercepts(s): $(-1.4, 0)$

y-intercept(s): $(0, 23)$

8. $y=-\frac{1}{2}(x-5)^3-2$

x	y
0	60.5
1	30
2	11.5
3	2
4	-1.5
5	-2
6	-2.5
7	-6
8	-15.5



Sign of the Lead Coefficient: negative

End behavior: ↖↘

Domain: all reals

Range: all reals

Relative minimum: none

Relative maximum: none

Interval(s) where function values are increasing: none

Interval(s) where function values are decreasing: $-\infty < x < \infty$

x-intercepts(s): $(3.4, 0)$

y-intercept(s): $(0, 60.5)$

9. How does a cubic function in the form $y=a(x-h)^3+k$, relate to the significant features of the graph?
"a" determines the end behavior; there is only one x-intercept; also with no rel min or rel max, the function values are either always increasing or always decreasing. So there are no turning points.

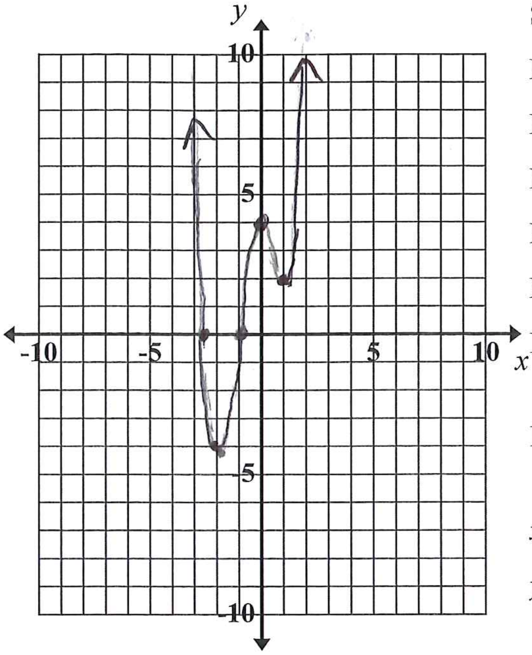
Name ______________________________ Period __________

6.1B *Graphing Polynomial Functions: Significant Features*

#10 – 11: Use a table of values to graph each equation and identify the significant features of the graph.

10. $y = x^4 + x^3 - 4x^2 + 4$

x	y
-3	22
-2	-4
-1	0
0	4
1	2
2	12
3	76



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: y ≥ -4.3

Relative minimum: (-1.9, -4.3), (1.1, 1.95)

Relative maximum: (0, 4)

Interval(s) where function values are increasing: -1.9 < x < 0, x > 1.1

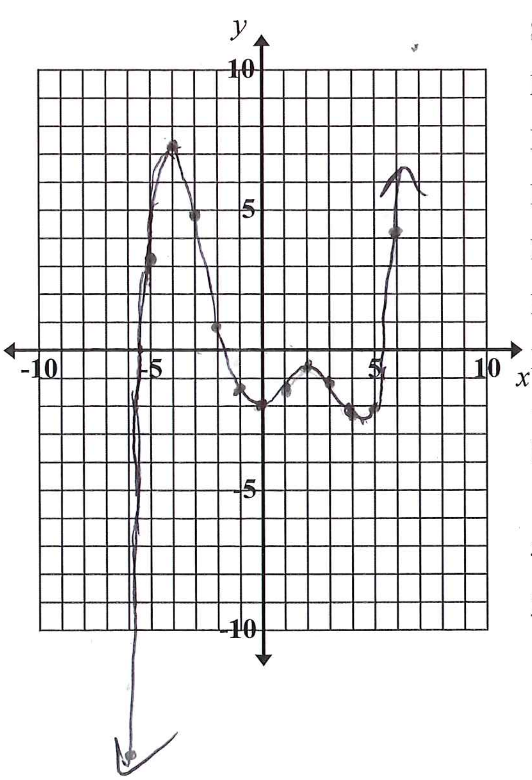
Interval(s) where function values are decreasing: x < -1.9, 0 < x < 1.1

x-intercepts(s): (-2.4, 0), (-1, 0)

y-intercept(s): (0, 4)

11. $y = \frac{1}{150}x^5 - \frac{1}{50}x^4 - \frac{1}{5}x^3 + \frac{3}{5}x^2 + \frac{3}{10}x - 2$

x	y
-6	-16.8
-5	3.2
-4	7.3
-3	4.7
-2	0.9
-1	-1.5
0	-2
1	-1.3
2	-0.7
3	-1.1
4	-2.3
5	-2.2
6	4.1



Sign of the Lead Coefficient: positive

End behavior: ↙ ↗

Domain: all reals

Range: all reals

Relative minimum: (-0.23, -2.03), (4.5, -2.6)

Relative maximum: (-4.1, 7.3), (2.2, -0.7)

Interval(s) where function values are increasing: x < -4.1, -.23 < x < 2.2, x > 4.5

Interval(s) where function values are decreasing: -4.1 < x < -.23, 2.2 < x < 4.5

x-intercepts(s): (-5.3, 0), (-1.7, 0), (5.5, 0)

y-intercept(s): (0, -2)

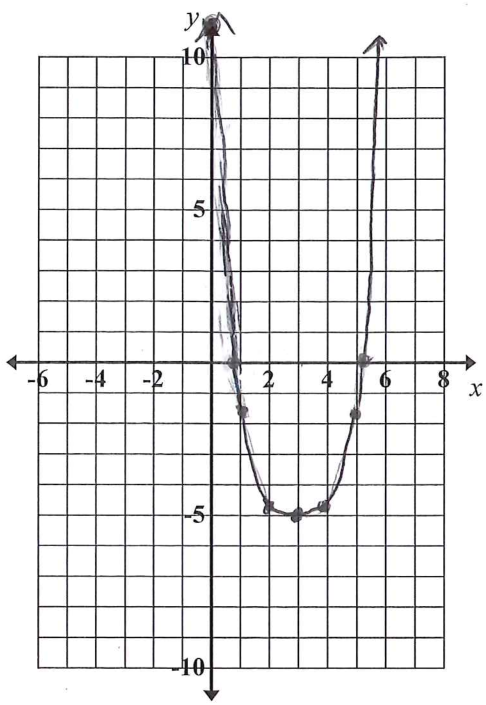
Name ______________________ Period ________

6.1B *Graphing Polynomial Functions: Significant Features*

#12 – 13: Use a table of values to graph each equation and identify the significant features of the graph.

12. $y=\frac{1}{5}(x-3)^4-5$

x	y
-1	46.2
0	11.2
1	-1.8
2	-4.8
3	-5
4	-4.8
5	-1.8
6	11.2
7	46.2



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: $y \geq -5$

Relative minimum: (3, -5)

Relative maximum: none

Interval(s) where function values are increasing: $x > 3$

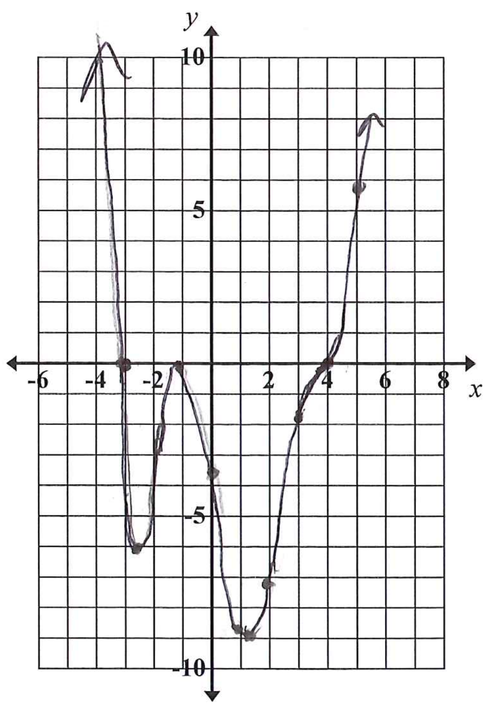
Interval(s) where function values are decreasing: $x < 3$

x-intercepts(s): (.8, 0), (5.2, 0)

y-intercept(s): (0, 11.2)

13. $y=\frac{1}{50}(x+3)(x+1)^2(x-4)^3$

x	y
-4	92.2
-3	0
-2	-4.32
-1	0
0	-3.8
1	-8.6
2	-7.2
3	-1.9
4	0
5	5.8



Sign of the Lead Coefficient: positive

End behavior: ↖ ↗

Domain: all reals

Range: $y \geq -8.96$

Relative minimum: (-2.45, -6.21), (1.29, -8.96)

Relative maximum: (-1, 0)

Interval(s) where function values are increasing: $-2.45 < x < -1$, $x > 1.29$

Interval(s) where function values are decreasing: $x < -2.45$, $-1 < x < 1.29$

x-intercepts(s): (-3, 0), (-1, 0), (4, 0)

y-intercept(s): (0, -3.84)

14. How does the degree of the polynomial function affect the end behavior of its graph?

Even degree: end beh is either ↖↗ or ↙↘

Odd degree: end beh is either ↙↗ or ↖↘

Name ______________________ Period ________

6.1B *Graphing Polynomial Functions: Significant Features*

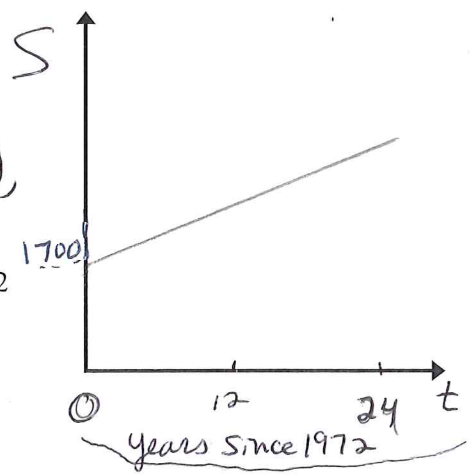
15. The retail space in shopping centers in the United States from 1972 to 1996 can be modeled by

$$S = -0.0068t^3 - 0.27t^2 + 150t + 1700$$

where S is the amount of retail space (in millions of square feet) and t is the number of years since 1972.

a) How much retail space was there in 1990? Record your thinking.

4272.9 ft^2 (millions)
$t=18$, since 1990 is 18 years since 1972



b) Is the amount of retail space increasing or decreasing in 1995? Record your thinking. Increasing; when $22 < t < 24$ (before and after 1995), S increases.

c) Is the amount of retail space decreasing at any point between 1972 and 1996? Record your thinking. NO
When $0 < t < 24$, S increases each year.

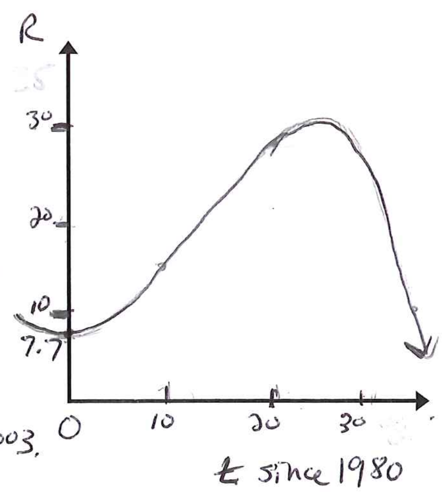
d) What are the domain and range of the function? What do they represent in the context of the problem?
Domain $0 \le t \le 24$, t is # years since 1972 up to 1996.
Range $1700 \le S \le 5050.5$; S is the retail space (in millions ft^2) over that time period.

16. The average monthly cable TV rate from 1980 to 2003 can be modeled by

$$R = -0.0036t^3 + 0.13t^2 - 0.073t + 7.7$$

where R is the monthly rate (in dollars) and t is the number of years since 1980.

a) What are the domain and range of the function? What do they represent in the context of the problem?
Domain $0 \le t \le 23$; # years from 1980 to 2003
Range $\$7.70 \le R \le \$30.99$; monthly rate for cable TV in that time period



b) Why isn't this model useful past 2003? (What would your cable bill be now if you used this model?) In the year 2003, the max. rate was \$31.06 (23.8, 31.06). After that, the rates decrease over time ... not the case! For 2014, this model would yield a rate of \$14/mo.!

c) When is the graph increasing? What does this mean in the context of the problem? $0 \le t \le 23$
Cable TV Rates were always increasing from 1980 to 2003.

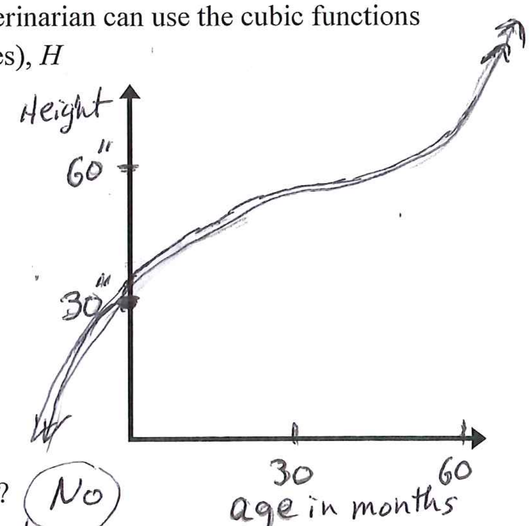
d) When is the graph decreasing? What does this mean in the context of the problem?
When $t > 23.8$ That the rates started declining in the year 2003.

e) What is the average monthly cost of a cable bill in 1983?
$R(3) = \$8.55$

Name ____________________ Period __________

6.1B Graphing Polynomial Functions: Significant Features

17. To determine whether a Holstein heifer's height is normal, a veterinarian can use the cubic functions included below, where L is the minimum normal height (in inches), H is the maximum normal height (in inches), and t is the age (in months).



$$L = 0.0007t^3 - 0.061t^2 + 2.02t + 30$$

$$H = 0.001t^3 - 0.08t^2 + 2.3t + 31$$

a) What is the normal height range for an 18 month old heifer? Record your thinking. $L(18) = 50.7$ in

$H(18) = 52.3$ in

$50.7'' <$ normal height $< 52.3''$

b) Can this model be used for the entire life of the heifer? Why? No

After a certain age, a heifer does not get any taller.

c) Suppose a veterinarian examines a Holstein heifer that is 43 inches tall. About how old do you think the heifer is? Explain how you got your answer. 6.6 months < age < 8.3 months

Draw a horizontal line $y = 43$ and use calc. intersect on Both curves.

$L(8.3) = 43$

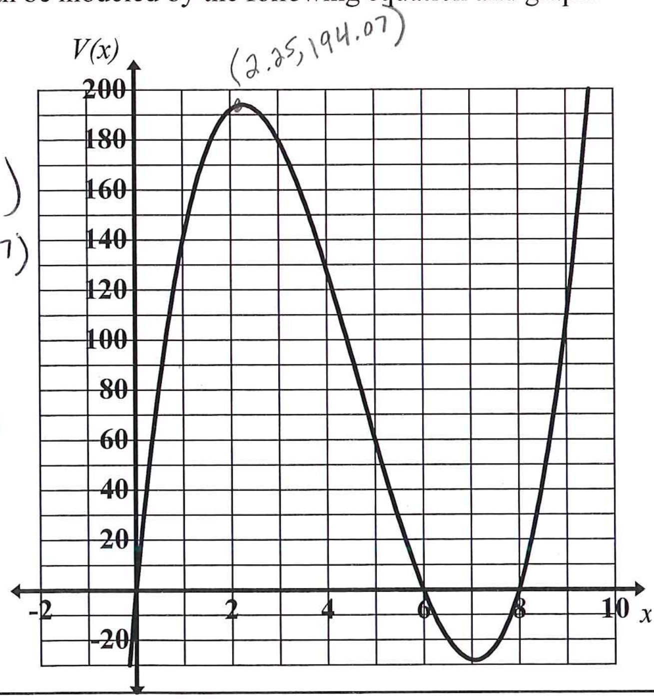
$H(6.6) = 43$

d) At what age does this model no longer work? (Hint: graph both functions at once)

At the point of inflection, the increasing height starts to level off before the graph starts to increase again. This occurs when the height is near 55", so probable age would be 30 months < age months < 31.6 months

18. A container company is making an open box from a 12 inch by 16 inch piece of metal, by cutting equal squares from each corner. The volume of the box can be modeled by the following equation and graph.

$$V(x) = (12 - 2x)(16 - 2x)(x)$$



a) Explain the restrictions on the domain and range of this model.

Domain $0 < x < 6$ (width > 0, $12 - 2x > 0 \Rightarrow x < 6$)

Range $0 < V < 194.07$ (volume > 0 and within given domain, the max Volume is 194.07)

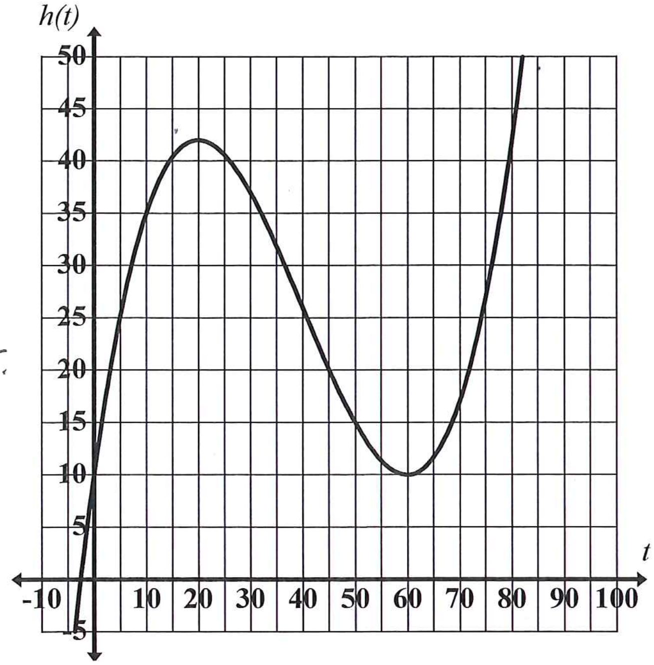
b) What is the approximate maximum volume for this box? Explain your thinking. 194 in^3

Within the acceptable domain $0 < x < 6$, the max Volume occurs at (2.26, 194.07)

c) What size squares should they cut off of each corner in order to maximize the volume of the box? (approximate) $\approx 2\frac{1}{4}''$ squares

6.1B Graphing Polynomial Functions: Significant Features

19. The function $h(t) = 0.001t^3 - 0.12t^2 + 3.6t + 10$ gives the height, h, (in feet) during the time t (in seconds) of a portion of the track of a roller coaster.



a) What is the y-intercept? What does this point represent in the context of the problem?

y-int = 10; The roller coaster is 10 ft above ground before it starts a certain portion of the track.

b) Is there a relative maximum? If yes, explain what it means in the context of the problem.

(20, 42) ⇒ after 20 seconds, the roller coaster reaches its max ht. of 42 feet.

c) Is there a relative minimum? If yes, explain what it means in the context of the problem.

(60, 10) ⇒ After 60 seconds, the roller coaster returns to a height of 10 feet.

d) Does the track ever touch the ground? How can you tell?

No; the height never equals 0 after the coaster begins rolling.

e) Use the graph to find $h(5)$. Explain what it means in the context of the problem.

h(5) = 25.1 After 5 seconds, the roller coaster has reached a height of 25.1 ft above the ground.

f) At what t-value is $h(t) = 35$? Explain what this means in the context of the problem.

At t = 10 secs and at t = 32.1 secs

The roller coaster's height of 35' is obtained twice during the ride — once on the way up and once again on its way down.

At t = 77, the height is again 35 feet, but the roller coaster ride lasts 60 seconds. 77 seconds is not part of the domain.

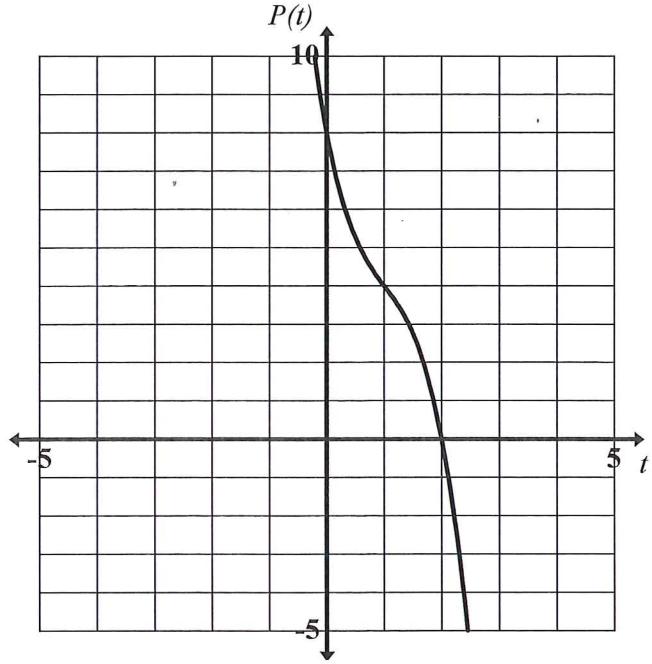
g) The ride lasts 60 seconds. Why would this model not be useful after 60 seconds?

After 60 seconds, the height of the coaster is continually increasing to ∞.

Name ______________________ Period ________

6.1B *Graphing Polynomial Functions: Significant Features*

20. A patient is receiving a certain medication in the hospital. The amount of drug (milligrams) in his bloodstream t days after the drug is taken can be modeled by the function $P(t) = -2t^3 + 6t^2 - 8t + 8$.



a) Use the graph to find how much of the drug was in the patient's bloodstream when his blood was tested 24 hours after he was given the drug.

4 mg

b) Tell the doctor how many hours it will take for the drug to be completely eliminated from his bloodstream.

2 days

c) Explain any restrictions on the domain and range for this model.

Domain $0 \le t \le 2$
After 2 days, the drug is completely out of the patient's bloodstream $(y=0)$

Range $0 \le P(t) \le 8$
8 mg is the original amount of the drug in the patient's bloodstream which lessens each hour afterwards.

Section 6.1B

Name ______________________________ Period __________

6.1B Graphing Polynomial Functions: Significant Features

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Name ______________________ Period __________

6.2A *Properties of Exponents*

#1 – 6: Multiple choice: Circle the correct answer

1. Which of the following is not equal to $\left(\frac{3}{4}\right)^0$? **[A]** 1^{23} **[B]** $\left(\frac{4}{3}\right)^0$ **[C]** 0^0	**2.** Simplify $3^2 \cdot 3^3$ **[A]** 3^5 **[B]** 3^6 **[C]** 9^5	**3.** Simplify $2^4 \cdot 2^3$ **[A]** 2^{12} **[B]** 2^7 **[C]** 4^7
4. Simplify $7^{12} \div 7^3$ **[A]** 7^4 **[B]** 7^9 **[C]** 7^{15}	**5.** Simplify $\left(3^2\right)^8$ **[A]** 3^{16} **[B]** 3^{10} **[C]** 3^6	**6.** Simplify $2^3 + 2^2$ **[A]** 10 **[B]** 12 **[C]** 32

7. True or False? If the equation is false, then correct it to make it true.

T **a)** $5x^2 \cdot \left(2x^3\right)^3 = 40x^{11}$ ✓

$5x^2 \cdot 8x^9 =$

$40x^{11}$ True

F **b)** $6x^2 + (3x)^2 = 9x^2$

$6x^2 + 9x^2 = 15x^2$

F **c)** $\left(2x^5z^4\right)^3 = 6x^{15}z^{12}$

$= 8x^{15}z^{12}$

8. Is the following statement true? $\left(x^a\right)^b = \left(x^b\right)^a$? Why or why not?

True

$x^{ab} = x^{ba}$, $ab = ba$ commutative property of multiplication

Using Power of a Power Rule)

9. Is the following statement true? $\left(x^2\right)^3 = x^{2+3}$? Why or why not?

False Using Power of a Power rule

$\left(x^2\right)^3 = x^{2\cdot3} = x^6$

Name ________________________________ Period __________

6.2A *Properties of Exponents*

#10 – 15: Simplify. Your answer should contain only positive exponents.

10. $4x^2x^3$

$4x^5$

11. $2k(3km)+4m(k^2)$

$6k^2m + 4k^2m$

$10k^2m$

12. $3x\cdot(2x^4)^3\cdot x^3$

$(3x)(8x^{12})\cdot x^3$

$24x^{16}$

13. $(3x^4y^5)^2$

$9x^8y^{10}$

14. $(2m^4)^3\cdot 2m^4$

$8m^{12}\cdot 2m^4$

$16m^{16}$

15. $(x^0)^4\cdot(2x^3)^3$

$(1)^4\cdot 8x^9$

$8x^9$

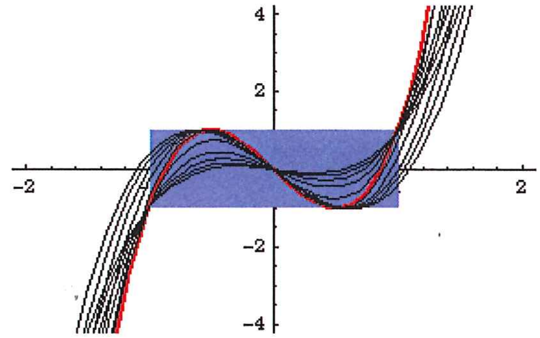
Section 6.2A

Name ______________________________ Period __________

6.2B *Polynomial Addition and Subtraction*

"Polynomials are temperamental creatures. If you force them to behave somewhere, they will go wild in other places."

-- Oved Shisha (1932-1998)



1. Jamal, Precious and Kiarra were working on the following problems during class. Did they do the problems correctly? If not, explain what they did wrong and fix their mistakes.

Jamal Incorrect

$$\begin{array}{r} 7x^3 - 3x^2 + 5 \\ +\ 2x^3 - 5x - 7 \\ \hline 9x^3 - 8x^2 - 2 \end{array}$$

These are not like terms.

$\boxed{9x^3-3x^2-5x-2}$

Precious Incorrect

$$\begin{array}{r} -3x^3 + 5x^2 + 4 \\ -\left(-8x^3 - 4x + 9\right) \\ \hline 5x^6 + 9x^3 - 5 \end{array}$$

These are not like terms; 2nd error on adding like terms, the exponent does not change from 3 to 6!

$\boxed{5x^3+5x^2+4x-5}$

Kiarra Incorrect

$$\left(4x^2 - 9x\right) - \left(-8x^2 + 3x - 7\right)$$
$$= \left(4x^2 + 8x^2\right) + \left(-9x + 3x\right) - 7$$
$$= 12x^2 - 6x - 7$$

2 sign errors when distributing

$(4x^2+8x^2)+(-9x-3x)+7$

$\boxed{12x^2-12x+7}$

#2 – 15: Find each sum or difference.

2. $\left(4x-5\right)+\left(3x+6\right)$

$\boxed{7x+1}$

3. $\left(3p^2-2p+3\right)-\left(p^2-7p+7\right)$

$\boxed{2p^2+5p-4}$

4. $\left(7x^2-8\right)+\left(3x^2+1\right)$

$\boxed{10x^2-7}$

5. $\left(x^2+y^2\right)-\left(-x^2+y^2\right)$

$2x^2$

6.
$$\begin{array}{r} 5a^2 + 3a^2x - 7a^3 \\ (+)\ 2a^2 - 8a^2x + 4a^3 \\ \hline \end{array}$$

$\boxed{7a^2-5a^2x-3a^3}$

7.
$$\begin{array}{r} 5x^2 - x - 4 \\ (-)\left(3x^2 + 8x - 7\right) \\ \hline \end{array}$$

$\boxed{2x^2-9x+3}$

Name ______________________________ Period __________

6.2B *Polynomial Addition and Subtraction*

#2 – 15 (continued): Find each sum or difference.

8.

$$\begin{array}{lr} & 2x+6y-3x+5 \\ & 4x-8y+6x-1 \\ (+) & x-3y \qquad +6 \\ \hline \end{array}$$

$= -x+6y+5$

$10x-8y-1$

$x-3y+6$

$\boxed{10x-5y+10}$

9.

$$\begin{array}{r} 11m^2n^2+2mn-11 \\ (-)(5m^2n^2-6mn+17) \\ \hline \end{array}$$

$\boxed{6m^2n^2+8mn-28}$

10. $(5x^2-x-7)+(2x^2+3x+4)$

$\boxed{7x^2+2x-3}$

11. $(5a+9b)-(4a+2b)$

$\boxed{a+7b}$

12. $(5x+3z)+9z$

$\boxed{5x+12z}$

13. $6p-(8q+5p)$

$\boxed{p-8q}$

14. $(5a^2x+3ax^2-5x)+(2a^2x-5ax^2+7x)$

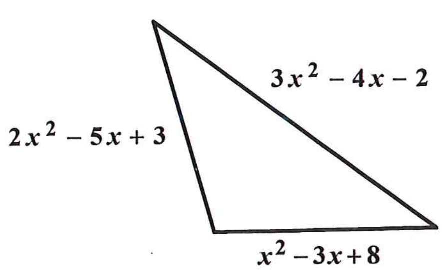
$\boxed{7a^2x-2ax^2+2x}$

15. $(x^3-3x^2y+4xy^2+y^3)-(7x^3-9x^2y+xy^2+y^3)$

$\boxed{-6x^3+6x^2y+3xy^2}$

16. Find the perimeter of the triangle.

$\boxed{P = 6x^2-12x+9}$



17. Find the difference between 5 more than the square of a number and 8 less than twice the number.

$(x^2+5)-(2x-8)$

$x^2+5-2x+8$

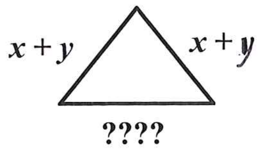
$\boxed{x^2-2x+13}$

Name ______________________ Period __________

6.2B *Polynomial Addition and Subtraction*

#18 – 19: Find the measure of the third side of each triangle. *P* is the measure of the perimeter.

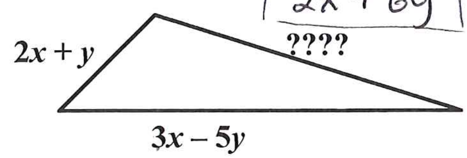
18. $P = 3x + 3y$



$\boxed{x + y}$

19. $P = 7x + 2y$

$\boxed{2x + 6y}$



20. Rectangular prism ***A*** has a volume of $x^3 + 2x^2 - 3$. Rectangular prism ***B*** has a volume of $x^4 + 2x^3 - 8x^2$. What is the difference when comparing the volume of rectangular prism ***B*** and the rectangular prism ***A***?

$$(x^4 + 2x^3 - 8x^2) - (x^3 + 2x^2 - 3) =$$

$$\boxed{x^4 + x^3 - 10x^2 + 3}$$

21. Suppose that two cars are having a race. The distance traveled by one car after t seconds is $10t^2 + 50t$ meters, while the distance traveled by the other car after t seconds is $15t^2 + 40t$ meters. How far would the two cars be apart after t seconds?

$$(15t^2 + 40t) - (10t^2 + 50t) = \boxed{5t^2 - 10t \text{ meters}}$$

22. At Anoka High School, the number of student tickets sold for a home football game can be modeled by $S(p) = 64p + 8450$ where p is the winning percent of the home team. The number of adult tickets sold for these home games is given by $A(p) = 0.5p^2 + 14p + 4200$. Write a function model representing the total number of tickets sold.

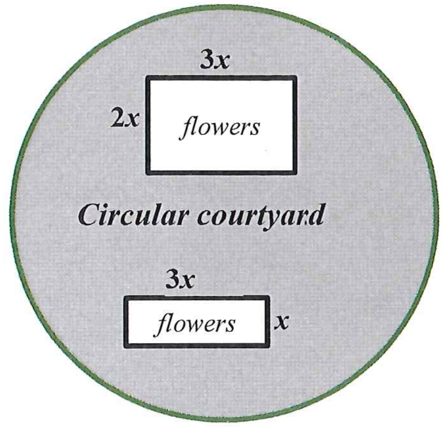
$$T(P) = S(p) + A(p)$$
$$= (64p + 8450) + (0.5p^2 + 14p + 4200)$$

$$\boxed{T(P) = 0.5p^2 + 78p + 12650}$$

Name ______________________ Period __________

6.2B *Polynomial Addition and Subtraction*

23. Andover High School is considering building a circular courtyard with 2 flower beds. The courtyard has an area of $10 - 2x^2$. Write and simplify an expression that represents the green lawn area.



$(10-2x^2)-6x^2-3x^2=$

$\boxed{-11x^2+10}$

24. While Jamal loves the wild rabbits in his back yard, he would like to keep the little critters out of his vegetable garden. If his vegetable garden is a rectangle whose width is represented by $5x^2 + 3x$ and whose length is represented by $2x^2 + 4$, how much fencing is needed to enclose the veggie garden?

$P = 2(5x^2+3x)+2(2x^2+4)$

$10x^2+6x+4x^2+8$

$\boxed{14x^2+6x+8}$

25. Find the missing term in each problem.

a) $(4x + \underline{\ 2\ }) + (2x+3) = 6x+5$

b) $(x^2+4x+8)-(3x^2+\underline{\ 5x\ }+6) = -2x^2-x+2$

c) $(4x + \underline{\ 8\ }) - (2x+3) = 2x+5$

d) $(x^2+6x+7)+(3x^2+\underline{\ 7x\ }+1) = 4x^2+13x+8$

e) $(4x + \underline{\ -1\ }) + (2x+3) - (x-1) = 5x+3$

Section 6.2B

Name ______________________ Period __________

6.2C *Polynomial Multiplication*

1. Thao, Louie and Monique were working on the following problems during class. Did they do the problems correctly or not? Explain what they did wrong and fix their mistakes.

Thao

$(x^2-7)(-3x^3+5x-4)$

$=-3x^6+5x^2-4x+21x^3-35x+28$

$=-3x^6+21x^3+5x^2-39x+28$

Used Product Property of Powers Incorrectly

$= -3x^5+5x^3-4x^2$
$+21x^3 - 35x + 28$

$= \boxed{-3x^5+26x^3-4x^2-35x+28}$

Louie

$(x+7)(x-3)$

$=x^2-10x-21$

Error: $-3x+7x=4x$

$\boxed{x^2+4x-21}$

Monique

$-3(2x^2-5x+7)$

$=-6x^2-5x+7$

Didn't distribute the -3 over the last 2 terms

$=\boxed{-6x^2+15x-21}$

#2 – 11: Find each product. Your final answer should be in standard form.

2. $4x(x^2-5x+8)$

$\boxed{4x^3-20x^2+32x}$

3. $-x^2(7x^3-13x+20)$

$\boxed{-7x^5+13x^3-20x^2}$

4. $(x^2-6x-7)(3x^2-9x+14)$

$= 3x^4-9x^3+14x^2$
$-18x^3+54x^2-84x$
$-21x^2+63x-98$

$= \boxed{3x^4-27x^3+47x^2-21x-98}$

5. $(x^2-5)(x+7)$

$\boxed{x^3+7x^2-5x-35}$

6. $(2x^2-4)(3x-4)$

$\boxed{6x^3-8x^2-12x+16}$

7. $(x+5)(x-2)(3x-7)$

$(3x-7)(x^2+3x-10) =$

$3x^3+9x^2-30x$
$-7x^2-21x+70$

$= \boxed{3x^3+2x^2-51x+70}$

Name ______________________ Period __________

6.2C Polynomial Multiplication

#2 – 11 (continued): Find each product. Your final answer should be in standard form.

8. $(6x+5)^2$

$\boxed{36x^2+60x+25}$

9. $(x^2-6x+15)(5x-4) =$

$5x^3-4x^2$
$-30x^2+24x$
$+75x-60$

$\boxed{5x^3-34x^2+99x-60}$

10. $2x^3(3x^3-x^2+12x-10)$

$\boxed{6x^6-2x^5+24x^4-20x^3}$

11. $(4x^3-x^2-3)(2x^2-x+6)$

$8x^5-4x^4+24x^3$
$-2x^4+x^3-6x^2$
$-6x^2+3x-18$

$\boxed{8x^5-6x^4+25x^3-12x^2+3x-18}$

#12 – 17: Find each product. Your final answer should be in standard form. Identify the *y*-intercept of the graph of the equation.

12. $y=(3x^2-4)(x^2+2)(x+3)$

$(3x^2-4)(x^3+3x^2+2x+6)$
$3x^5+9x^4+6x^3+18x^2$
$-4x^3-12x^2-8x-24$

$\boxed{y=3x^5+9x^4+2x^3+6x^2-8x-24}$

y-intercept: -24

13. $y=(3x-7)^2$

$\boxed{y=9x^2-42x+49}$

y-intercept: 49

14. $y=3(-2x+15)(4x^2-1)$

$3(-8x^3+2x+60x^2-15)$

$\boxed{y=-24x^3+180x^2+6x-45}$

y-intercept: -45

15. $y=(x^2+1)(x^2-4x+11)$

$x^4-4x^3+11x^2$
$+1x^2-4x+11$

$y=x^4-4x^3+12x^2-4x+11$

y-intercept: 11

16. $y=(8-x^2)(x+3)$

$=8x+24-x^3-3x^2$

$y=-x^3-3x^2+8x+24$

17. $y=(5x-3)(x^2-7x+11)$

$=5x^3-35x^2+55x$
$-3x^2+21x-33$

$y=5x^3-38x^2+76x-33$

Name ______________________ Period __________

6.2C Polynomial Multiplication

y-intercept: 24   *y*-intercept: $^{-}33$

18. What information about the graph of a polynomial is easily found when the equation is in standard form?

a) The end behavior is determined by looking at the degree (the highest exponent which should be the 1st term) and the leading coefficient.
b) The y-intercept is the constant, or last term in standard form.

19. The average amount of bananas (in pounds) eaten per person each year in the United States from 1995 to 2000 can be modeled by $f(x) = -0.298x^3 - 2.73x^2 + 7.05x + 78.45$ where x is the number of years since 1995. Graph the function using a graphing utility.

a) What is the *y*-intercept? Explain the meaning in the context of the problem.

78.45; In 1995, the average amount of bananas (in pounds) eaten per person in the US was 78.5

b) Is this function increasing or decreasing? Explain the meaning of this in the context of the problem.

Increasing from 1995 to 1996, then decreasing amount of bananas consumed from 1996 to 2000.

20. From 2005 through 2013, the number of paperback books N (in millions) sold in the United states and the average price per book P (in dollars) can be modeled by $N(t) = 1.36t^2 + 2.53t + 1076$ and $P(t) = 0.314t + 3.42$ where t is the number of years since 2005.

a) Write a function for the total revenue (amount of money made) R received from the sales of paperback books.

$R(t) = (0.314t + 3.42)(1.36t^2 + 2.53t + 1076)$

$0.427t^3 + 0.794t^2 + 337.864t$

$4.651t^2 + 8.653t + 3679.92$

$R(t) = 0.427t^3 + 5.445t^2 + 346.517t + 3679.92$

b) What was the total revenue from paperback books in 2005?

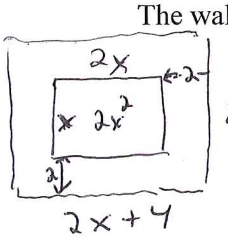
\$3679.92 millions or

\$3,679,920,000

c) What point on the graph represents this value?

y-intercept

21. A rectangular swimming pool is twice as long as it is wide. A small concrete walkway surrounds the pool. The walkway is a constant 2 feet wide and has an area of 196 square feet. Find the dimensions of the pool.



A(pool + sidewalk) − A pool = A sidewalk

$(x+4)(2x+4) - 2x^2 = 196$

$2x^2 + 12x + 16 - 2x^2 =$

$12x + 16 = 196$

width $x = 15$ feet
&
length $2x = 30$ feet
of pool

Name ______________________________ Period __________

6.2C Polynomial Multiplication

22. The revenue (revenue = profit – cost) in dollars from the sale of scooters can be represented by $R(x) = (-x^2 + 6000)(x - 40)$ where x is the number of scooters sold.

a) Put the equation in standard form.

$R(x) = -x^3 + 40x^2 + 6000x - 240{,}000$

b) If they do not sell any scooters, what is the revenue?

$R(0) = \$-240{,}000$

c) Does this number make sense? Why?

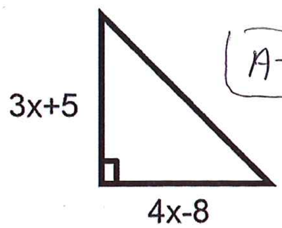
yes; negative # for revenue means the costs were higher than the sales (which was $0.)

23. Write an expression for the area of the triangle. Simplify the expression completely.

$A = \frac{1}{2}(4x-8)(3x+5)$

$(2x-4)(3x+5)$

$\boxed{A = 6x^2 - 2x - 20}$



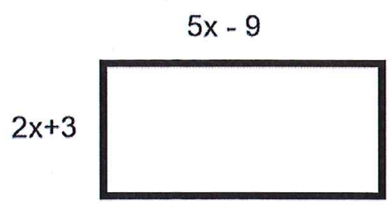
$A = \frac{1}{2} b \cdot h$

24. Write an expression for the product of 3 more than 4 times the square of a number and 7 less than five times the number. Simplify.

$(4x^2+3)(5x-7) =$

$\boxed{20x^3 - 28x^2 + 15x - 21}$

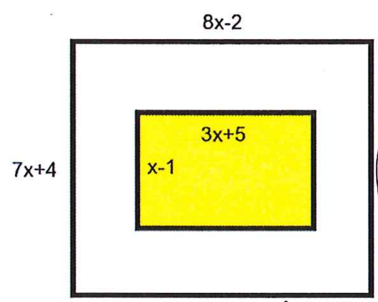
25. Write an expression for the area of the box. Simplify.



$(5x-9)(2x+3) =$

$\boxed{10x^2 - 3x - 27}$

26. Write an expression for the area of the part of the floor not covered by the rug. Simplify.



$A_{\text{nonshaded}}$

$53x^2 + 16x - 3$

$A = (8x-2)(7x+4) - (3x+5)(x-1)$

$(56x^2 + 18x - 8) - (3x^2 + 2x - 5) =$

27. Think of a number. Subtract 7. Multiply by 3. Add 30. Divide by 3. Subtract the original number. The result is always 3. Use a polynomial equation to illustrate this number trick.

$\frac{3(n-7)+30}{3} - n = 3$

$\frac{3n - 21 + 30}{3} - n$

$n - 7 + 10 - n$

$3 = 3$ ✓

Name ______________ Period ________

6.2C *Polynomial Multiplication*

28. Find the missing term.

a) $(x+5)(x+\underline{\ 3\ })=x^2+8x+15$

b) $(\underline{\ 2x\ })(x^2+3x-7)=2x^3+6x^2-14x$

c) $(x-6)(x+\underline{\ ^-7\ })=x^2 - 13x + 42$

d) $(x+\underline{\ 3\ })(x^2+2x+4)=x^3+5x^2+10x+12$

29. The side of a cube is represented by $x+1$. Write an expression for the volume of the cube. Simplify.

$(x+1)^3 = (x+1)(x+1)(x+1)$

$(x+1)(x^2+2x+1) = x^3+2x^2+x$

x^2+2x+1

x^3+3x^2+3x+1

30. Let an integer be represented by x. Write an expression for the product of three consecutive integers starting with x. Simplify.

$x(x+1)(x+2)$

$x(x^2+3x+2) = \boxed{x^3+3x^2+2x}$

31. $N(f)$ represents the number of bags of chips that are sold when the school store has f flavors available for sale and $P(f)$ is the price, in dollars, of a bag of chips when there are f flavors available for sale. Write a sentence explaining what $N(f)\cdot P(f)$ means.

The #bags of various flavored chips times the price/bag gives the amount of money collected from the sales.

Section 6.2C

6.2C Polynomial Multiplication

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Name ______________________________ Period __________

6.2D *Polynomial Division – Part I*

#1 – 5: Find each quotient using polynomial long division and state if the binomial is a factor.

1. $\left(2x^4+15x^3-30x^2-20x+63\right)\div(x+9)$

$$2x^3-3x^2-3x+7$$

$x+9 \,\big)\, 2x^4+15x^3-30x^2-20x+63$

$-(2x^4+18x^3)$

$-3x^3-30x^2$

$-(-3x^3-27x^2)$

$-3x^2-20x$

$-(-3x^2-27x)$

$+7x+63$

$-(7x+63)$

0

yes, $(x+9)$ is a factor.

2. $\left(5x^5-3x^4+2x^3-30x^2-7x+3\right)\div(x-2)$

$$5x^4+7x^3+16x^2+2x-3-\frac{3}{x-2}$$

$x-2 \,\big)\, 5x^5-3x^4+2x^3-30x^2-7x+3$

$-(5x^5-10x^4)$

$7x^4+2x^3$

$-(7x^4-14x^3)$

$16x^3-30x^2$

$-(16x^3-32x^2)$

$2x^2-7x$

$-(2x^2-4x)$

$-3x+3$

$-(-3x+6)$

-3

not a factor

3. $\left(4x^2-5\right)\div(2x+6)$

$$2x-6+\frac{31}{2x+6}$$

$2x+6 \,\big)\, 4x^2+0x-5$

$-(4x^2+12x)$

$-12x-5$

$-(-12x-36)$

31

not a factor

4. $\left(-4x^6-5x^3+3x^2+x+7\right)\div(x-1)$

$$-4x^5-4x^4-4x^3-9x^2-6x-5+\frac{2}{x-1}$$

$x-1 \,\big)\, -4x^6+0x^5+0x^4-5x^3+3x^2+x+7$

$-(-4x^6+4x^5)$

$-4x^5+0x^4$

$-(-4x^5+4x^4)$

$-4x^4-5x^3$

$-(-4x^4+4x^3)$

$-9x^3+3x^2$

$-(-9x^3+9x^2)$

$-6x^2+x$

$-(-6x^2+6x)$

$-5x+7$

$-(-5x+5)$

2

not a factor

5. $\left(2x^3-5x^2+6x-2\right)\div(2x-1)$

$$x^2-2x+2$$

$2x-1 \,\big)\, 2x^3-5x^2+6x-2$

$-(2x^3-x^2)$

$-4x^2+6x$

$-(-4x^2+2x)$

$4x-2$

$-(4x-2)$

0

yes, $(2x-1)$ is a factor of $(2x^3-5x^2+6x-2)$

Name ______________________ Period __________

6.2D *Polynomial Division – Part I*

#6 – 9: Find each quotient using synthetic division and state if the binomial is a factor.

6. $\left(x^3+6x^2+7x+10\right)\div(x+2)$

-2	1	6	7	10
		-2	-8	2
	1	4	-1	12

$x^2+4x-1+\frac{12}{x+2}$

Not a factor

7. $\dfrac{4x^3-15x^2-120x-128}{x-8}$

8	4	-15	-120	-128
		32	136	128
	4	17	16	0

$4x^2+17x+16$

$(x-8)$ is a factor of $(4x^3-15x^2-120x-128)$.

8. $\left(3x^5+4x^3-x-2\right)\div(x-1)$

1	3	0	4	0	-1	-2
		3	3	7	7	6
	3	3	7	7	6	4

$\boxed{3x^4+3x^3+7x^2+7x+6+\frac{4}{x-1}}$

Not a factor

9. $\dfrac{x^3-3x^2-11x+5}{x-5}$

5	1	-3	-11	5
		5	10	-5
	1	2	-1	0

x^2+2x-1

$(x-5)$ is a factor of $(x^3-3x^2-11x+5)$

10. Which of the division problems above (#1 – 9) generate no remainder? What does that mean regarding the relationship of the polynomials if no remainder occurs when dividing?

#7 and #9; the divisor is a factor of the dividend.

11. Lia, Maut and Craig were working on the following problems during class. Did they do the problems correctly? If not, explain what they did wrong and fix their mistakes.

<u>Lia</u>

$\left(x^3+2x^2-6x-9\right)\div(x-2)$

2	1	2	-6	-9
		2	8	4
	1	4	2	-5

Correct, except to write the quotient:

$\boxed{x^2+4x+2-\frac{5}{x-2}}$

<u>Maut</u>

$\dfrac{10x^4+5x^3+4x^2-9}{x+1}$

-1	10	5	4	-9
		-10	5	-9
	10	-5	9	-18

missing a place holder for the x term.

-1	10	5	4	0	-9
		-10	5	-9	9
	10	-5	9	-9	0

$\boxed{10x^3-5x^2+9x-9}$

<u>Craig</u>

$\left(x^2-4x+3\right)\div(x-2)$

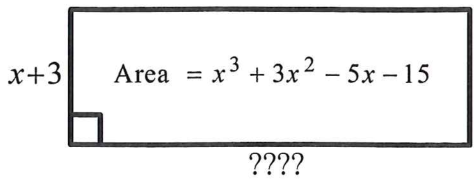
-2	1	-4	3
		-2	12
	1	-6	15

2	1	-4	3
		2	-4
	1	-2	-1

$x-2-\frac{1}{x-2}$

6.2D *Polynomial Division – Part I*

12. Find the length of the rectangular garden.



$$\begin{array}{r|rrrr} -3 & 1 & 3 & -5 & -15 \\ & & -3 & 0 & 15 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$(x^2 - 5)$ is the length

W · L = Area

$(x+3)(????) = x^3 + 3x^2 - 5x - 15$

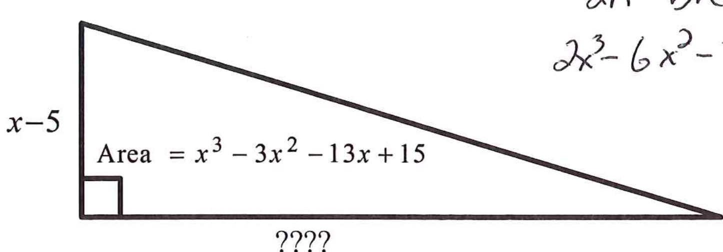
so if we divide $\dfrac{x^3 + 3x^2 - 5x - 15}{x+3}$

we can find the length (????)

13. Find the length of the base of the triangle below if $A = \frac{1}{2}bh$.

$2A = bh$

$2x^3 - 6x^2 - 26x + 30 = (\text{base})(x-5)$



$$\begin{array}{r|rrrr} 5 & 2 & -6 & -26 & 30 \\ & & 10 & 20 & -30 \\ \hline & 2 & 4 & -6 & 0 \end{array}$$

$(2x^2 + 4x - 6)$ is the Base

14. Suppose that you know that the area of a rectangular mural (wall painting) in square feet is represented by the polynomial $x^2 + 2x - 24$ and that the length of the mural in feet if the length is represented by the binomial $x + 6$. How would you calculate the width of the mural? Would it also be a binomial?

Divide the area by the length (Synthetically) to get the width.
Yes, the width would also be a binomial.

$$\begin{array}{r|rrr} -6 & 1 & 2 & -24 \\ & & -6 & 24 \\ \hline & 1 & -4 & 0 \end{array}$$

$(x-4)$ is the width.

Name ______________________ Period __________

6.2D *Polynomial Division – Part I*

15. If **A** and **B** are polynomials and **A** divided by **B** equals $5x^2 - 13x + 47 - \frac{102}{x+2}$.

a) Find **B**. $(x+2)$

b) Describe what you did to find this. B is the divisor, located in the denominator of the remainder fraction.

c) Find **A**.

$$\frac{A}{B} = (5x^2 - 13x + 47) - \frac{102}{x+2}$$

$$B \cdot \frac{A}{B} = B \cdot \left[(5x^2 - 13x + 47) - \frac{102}{x+2}\right]$$

$$A = (x+2)(5x^2 - 13x + 47) - 102$$

$$= 5x^3 - 13x^2 + 47x$$
$$+ 10x^2 - 26x + 94 \quad - 102$$

$$A = 5x^3 - 3x^2 + 21x - 8$$

16. Write a polynomial division problem where the use of synthetic division would be an appropriate strategy to use. Divide the polynomial problem you have written to find the quotient and remainder (if there is one).

one sample $(2x^3 - x + 3) \div (x - 1)$

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -1 & 3 \\ & & 2 & 2 & 1 \\ \hline & 2 & 2 & 1 & 4 \end{array}$$

$$2x^2 + 2x + 1 + \frac{4}{x-1}$$

17. Write a polynomial division problem which you **cannot** use synthetic division to simplify. Explain your reasoning why synthetic division cannot be used. Divide the polynomial expression you have written to find the quotient and remainder (if there is one).

one sample $(x^3 - 3x^2 + x - 3) \div (x^2 + 1)$

Synthetic Division can only be used when the divisor is a binomial with degree 1, like $(x - c)$

Using long division,

$(x - 3)$ is the quotient.

$$\begin{array}{r} x^2+1 \overline{\big)\, x^3 - 3x^2 + x - 3} \\ -(x^3 \qquad\quad + x) \\ \hline -3x^2 \qquad - 3 \\ -(-3x^2 \qquad - 3) \\ \hline 0 \end{array}$$

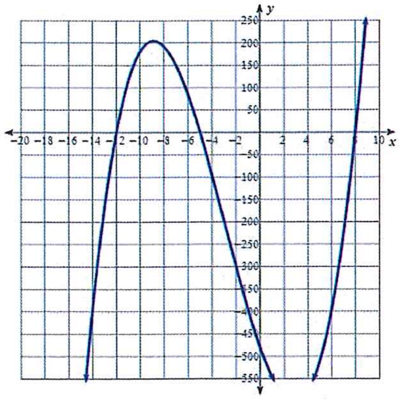
Section 6.2D

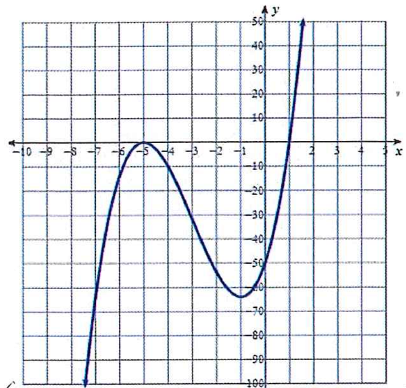
Name ______________________________ Period __________

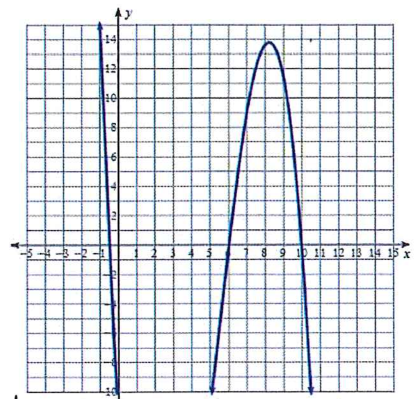
6.2E *Polynomial Division – Part II*

1. Look at each equation and its graph. What does the factored form of a polynomial tell us about the graph of that polynomial?

a) $f(x)=(x+5)(x-8)(x+12)$ **b)** $f(x)=2(x+5)^2(x-1)$ **c)** $f(x)=-\frac{1}{5}(2x+1)(x-10)(x-6)$







There are 3 distinct linear factors, which yield 3 x-intercepts at −12, −5, and 8

There is one duplicate linear factor which gives a double root at x = −5 (but only one x-int here), and the other linear factor yields the x-intercept at x = 1

There are 3 distinct linear factors, which yield 3 x-intercepts at x = −½, x = 10, x = 6

2. Name 2 other ways to say "x-intercept".

zero, root, solution

3. Explain how to find the zero of a polynomial function $f(x)$ both algebraically and graphically?

algebraically: set the function equal to 0 and solve for x

graphically: Look for the x-intercept(s) and/or use a graphing calculator to "calculate the zeros".

4. Is $3x^3-4x^2+x+2$ divisible by $x-1$? Explain your thinking. No; there is a remainder.

$$\begin{array}{r|rrrr} 1 & 3 & -4 & 1 & 2 \\ & & 3 & -1 & 0 \\ \hline & 3 & -1 & 0 & 2 \end{array}$$

$3x^2-x+\frac{2}{x-1}$

5. Is $x-1$ a factor of $x^{200}+1$? Explain your thinking.

No, there is a remainder of 2. If (x−1) was a factor, there would be a zero remainder.

$$\begin{array}{r|l} 1 & 1\ 0\ 0\ 0\ 0\ 0 \cdots\ 1 \\ & \ \ 1\ 1\ 1\ 1\ 1 \cdots\ 1 \\ \hline & 1\ 1\ 1\ 1\ 1\ 1\ \ 1\ \ 2 \end{array}$$

Name ______________________ Period __________

6.2E Polynomial Division – Part II

6. Is $x = -4$ a root (zero/solution) of $P(x) = x^3 + 14x^2 + 5x - 140$? Explain your thinking. $\boxed{\text{Yes}}$

$$\begin{array}{r|rrrr} -4 & 1 & 14 & 5 & -140 \\ & & -4 & -40 & 140 \\ \hline & 1 & 10 & -35 & 0 \end{array}$$

Yes, a zero remainder means $(x+4)$ is a factor of the polynomial, which means $x=-4$ is a zero or solution!

7. Is $x = 3$ a zero of $P(x) = 2x^3 - 7x^2 + 5x - 1$?

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 5 & -1 \\ & & 6 & -3 & 6 \\ \hline & 2 & -1 & 2 & 5 \end{array}$$

$\boxed{\text{No}}$

8. Find k so that $x-3$ is a factor of $3x^3 + 2kx^2 + (k+2)x - 3$.

$$\begin{array}{r|rrrr} 3 & 3 & 2k & k+2 & -3 \\ & & 9 & 6k+27 & 3 \\ \hline & 3 & 2k+9 & 1 & 0 \end{array}$$

must be "0" because x-3 is a factor. Therefore, value must be 3 to create that 0 remainder, which means value mult. by (outside) 3 must be 1.

$(k+2) + (6k+27) = 1$

$7k + 29 = 1$

$7k = -28$

$\boxed{k = -4}$

9. Find k so that $x-2$ is a factor of $f(x) = 3x^3 + 4x^2 + kx - 19x - 2$.

$$\begin{array}{r|rrrr} 2 & 3 & 4 & (k-19) & -2 \\ & & 6 & 20 & 2^{**} \\ \hline & 3 & 10 & 1^{***} & 0^{*} \end{array}$$

$k - 19 + 20 = 1$

$k + 1 = 1$

$\boxed{k = 0}$

* must be zero

** must be 2 to create 0 value

*** must be 1 to be able to mult & create value of 2

10. Write a 3rd degree equation of a polynomial function with the zeroes: 0, 2, and –5. Write your answer in factored form.

$f(x) = 2x(x-2)(x+5)$

11. Write a 3rd degree polynomial function with the zeroes: –2, 2, and 6. Write your answer in standard form.

$p(x) = (x+2)(x-2)(x-6)$

$= (x^2-4)(x-6)$

$\boxed{p(x) = x^3 - 6x^2 - 4x + 24}$

check:

$3x^3 + 2kx^2 + (k+2)x - 3$

(believe $k = -4$)

$3x^3 + 2(-4)x^2 + (-4+2)x - 3$

$3x^2 - 8x^2 - 2x - 3$ is $(x-3)$ a factor?

$$\begin{array}{r|rrrr} 3 & 3 & -8 & -2 & -3 \\ & & 9 & 3 & 3 \\ \hline & 3 & 1 & 1 & 0 \end{array}$$

yes, $k = -4$

Name ____________________ Period ________

6.2E *Polynomial Division – Part II*

12. Given a factor, write the function in factored form.

a) $(x-5);\ y = x^3 - 2x^2 - 13x - 10$

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$x^2 + 3x + 2 =$

$\boxed{y = (x-5)(x+2)(x+1)}$

b) $(x+1);\ y = x^3 - 9x^2 + 15x + 25$

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 15 & 25 \\ & & -1 & 10 & -25 \\ \hline & 1 & -10 & 25 & 0 \end{array}$$

$x^2 - 10x + 25 =$

$\boxed{y = (x+1)\cdot(x-5)(x-5)}$

c) $(x+2);\ y = x^3 + 3x^2 - 4$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & 0 & -4 \\ & & -2 & -2 & 4 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$x^2 + x - 2$

$(x+2)\cdot(x+2)(x-1)$

$\boxed{y = (x+2)^2(x-1)}$

d) $(x+1);\ y = x^3 + x^2 - 25x - 25$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -25 & -25 \\ & & -1 & 0 & 25 \\ \hline & 1 & 0 & -25 & 0 \end{array}$$

$x^2 - 25$

$\boxed{y = (x+1)\cdot(x+5)(x-5)}$

13. Given one zero, write the function in factored form.

a) $f(x) = x^3 + 2x^2 - 5x - 6;\ -3$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$(x^2 - x - 2) =$

$\boxed{f(x) = (x-2)(x+1)(x+3)}$

b) $f(x) = x^3 - 4x^2 - 17x + 60;\ 5$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & -17 & 60 \\ & & 5 & 5 & -60 \\ \hline & 1 & 1 & -12 & 0 \end{array}$$

$x^2 + x - 12 =$

$\boxed{f(x) = (x+4)(x-3)(x-5)}$

c) $f(x) = 9x^3 + 10x^2 - 17x - 2;\ -2$

$$\begin{array}{r|rrrr} -2 & 9 & 10 & -17 & -2 \\ & & -18 & 16 & 2 \\ \hline & 9 & -8 & -1 & 0 \end{array}$$

$9x^2 - 8x - 1 =$

$\boxed{f(x) = (9x+1)(x-1)(x+2)}$

d) $f(x) = 2x^3 + 3x^2 - 39x - 20;\ 4$

$$\begin{array}{r|rrrr} 4 & 2 & 3 & -39 & -20 \\ & & 8 & 44 & 20 \\ \hline & 2 & 11 & 5 & 0 \end{array}$$

$2x^2 + 11x + 5 =$

$\boxed{f(x) = (2x+1)(x+5)(x-4)}$

e) $f(x) = 3x^3 + 19x^2 + 16x - 20;\ -2$

$$\begin{array}{r|rrrr} -2 & 3 & 19 & 16 & -20 \\ & & -6 & -26 & 20 \\ \hline & 3 & 13 & -10 & 0 \end{array}$$

$3x^2 + 13x - 10 =$

$\boxed{f(x) = (3x-2)(x+5)(x+2)}$

f) $f(x) = 5x^3 + 11x^2 - 13x - 3;\ -3$

$$\begin{array}{r|rrrr} -3 & 5 & 11 & -13 & -3 \\ & & -15 & 12 & 3 \\ \hline & 5 & -4 & -1 & 0 \end{array}$$

$5x^2 - 4x - 1 =$

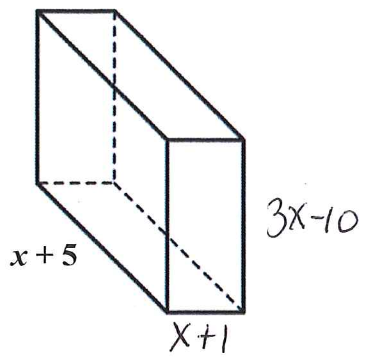
$\boxed{f(x) = (5x+1)(x-1)(x+3)}$

Name ______________________ Period __________

6.2E *Polynomial Division – Part II*

14. You are given an expression for the volume of the rectangular prism $\left(V = l \cdot w \cdot h\right)$. Find an expression for each of the missing dimensions.

a) $V(x) = 3x^3 + 8x^2 - 45x - 50$



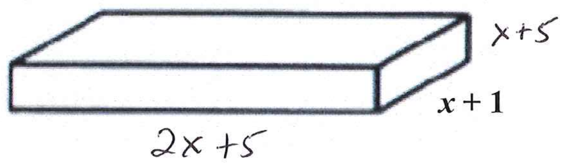
$$\begin{array}{r|rrrr} -5 & 3 & 8 & -45 & -50 \\ & & -15 & 35 & 50 \\ \hline & 3 & -7 & -10 & 0 \end{array}$$

$3x^2 - 7x - 10 =$

$(3x-10)(x+1)$

height · width

b) $V(x) = 2x^3 + 17x^2 + 40x + 25$



$$\begin{array}{r|rrrr} -1 & 2 & 17 & 40 & 25 \\ & & -2 & -15 & -25 \\ \hline & 2 & 15 & 25 & 0 \end{array}$$

$2x^2 + 15x + 25 =$

$(2x+5)(x+5)$

length · height

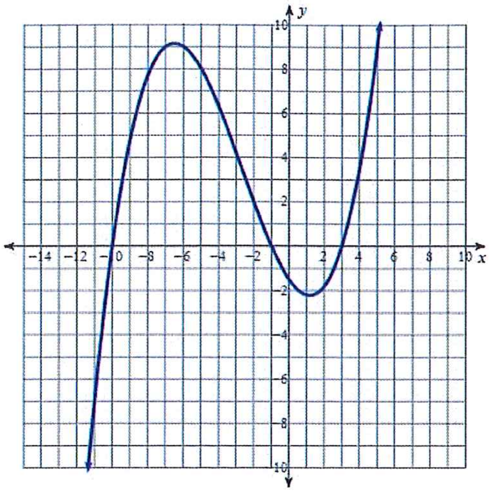
Section 6.2E

Name ______________________________ Period __________

6.3A Using Graphs to Find Solutions of Cubic Equations

1. Use the graph to find the zeros of each function.

a) **Graph:**



Real zeros:

$x = -10, -1, 3$

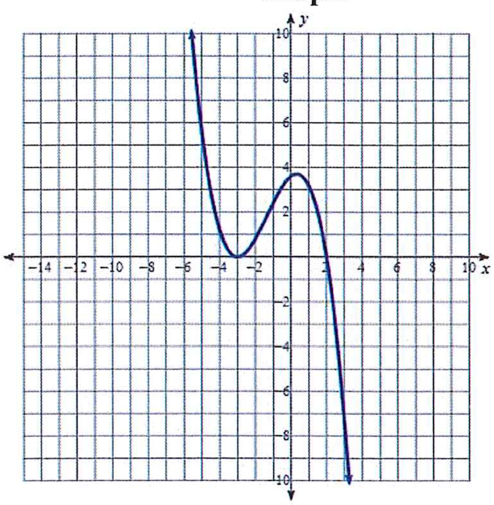
Factor(s) that create the zeros:

$(x+10)(x+1)(x-3)$

Possible equation of the curve to the left:

$y = \frac{1}{20}(x+10)(x+1)(x-3)$

b) **Graph:**



Real zeros:

$x = -3, 2$

Double Root

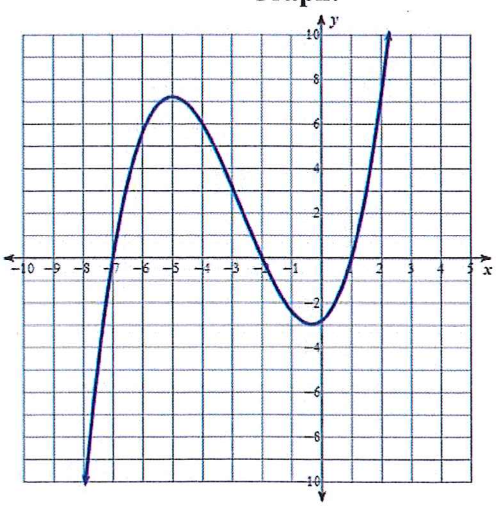
Factor(s) that create the zeros:

$(x+3)(x+3)(x-2)$

Possible equation of the curve to the left:

$y = -\frac{1}{3}(x+3)(x+3)(x-2)$

c) **Graph:**



Real zeros:

$x = -7, -2, 1$

Factor(s) that create the zeros:

$(x+7)(x+2)(x-1)$

Possible equation of the curve to the left:

$y = \frac{1}{5}(x+7)(x+2)(x-1)$

Name ______________________ Period ________

6.3A *Using Graphs to Find Solutions of Cubic Equations*

2. Using a graphing utility, use the table of values and/or the graph to find the x-intercepts. If necessary, round your answers to the nearest thousandth.

a) $y = x^3 - 8x^2 + 19x - 12$

(1,0)
(3,0)
(4,0)

b) $y = x^3 + 2x^2 - 12x + 10$

(-4.879, 0)
(1.287, 0)
(1.592, 0)

c) $g(x) = x^3 - 14x^2 + 47x - 18$

(0.438, 0)
(4.562, 0)
(9, 0)

d) $h(x) = x^3 + x^2 + 2x + 24$

(-3, 0)

3. Using a graphing utility, use the table of values and/or the graph to find the solutions to the equation $f(x) = 0$.

a) $f(x) = 3x^3 - 7x^2 + 8x - 2$

$x = \frac{1}{3}$

b) $f(x) = -4x^3 - 7x^2 + 4x + 3$

$x = -2.059, -0.469,$ or 0.777

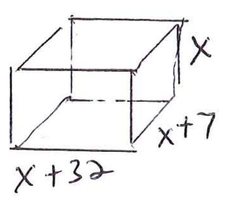
c) $f(x) = -x^3 + 2x^2 + 5x - 6$

$x = -2, 1,$ or 3

d) $f(x) = x^3 - 3x^2 + 4$

$x = -1$ or 2

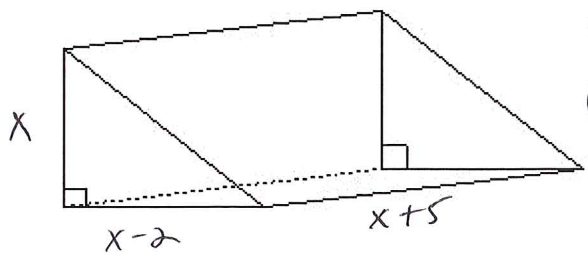
4. You are designing a swimming pool with a volume of 4800ft^3. The width of the pool should be 7 feet more than the depth, and the length should be 32 more feet than the depth. What should the dimensions of the pool be? (draw a sketch of the situation)



$x(x+7)(x+32) = 4800$
$x(x^2+39x+224)$
$x^3 + 39x^2 + 224x - 4800 = 0$
$x = 8$, so
height = 8 ft
width = 15 ft
length = 40 ft

6.3A *Using Graphs to Find Solutions of Cubic Equations*

5. You are building a solid concrete skate board ramp. The width of the ramp is 2 feet less than the height of the ramp and the length of the ramp platform is 5 feet more than the height of the ramp. If 420 cubic feet of concrete is used, what are the dimensions of the ramp?



$(AB)\cdot h = V$

$(\frac{1}{2}bh)\cdot h = 420$

$(\frac{1}{2}x(x-2))\cdot(x+5) = 420$

$2(\frac{1}{2}x(x-2)(x+5)) = 420(2)$

$x(x-2)(x+5) = 840$

$(x^2-2x)(x+5)$

$x^3+3x^2-10x-840 = 0$

$x \approx 8.85$

height = 8.85'
width = 6.85'
length = 13.85'

6. A car dealership's profit can be modeled by the function $P(x) = x^3 + 2x^2 + 400x - 400$, where x is the number of cars. How many cars will they have to sell to make $40,000 profit?

$40,000 = x^3+2x^2+400x-400$

$0 = x^3+2x^2+400x-40,400$

$x \approx 29.9$ so $\boxed{\text{Sell 30 cars}}$ to make a $40,000 profit.

7. The volume of a box can be modeled by $V(x) = 144x - 48x^2 + 4x^3$ where x is measured in meters and $V(x)$ is measured in ~~meters~~. Find the values of x that make $V(x) = 0$.
cubic meters.

$x = 0$ or $x = 6$

Section 6.3A

Name ______________________ Period __________

6.3A Using Graphs to Find Solutions of Cubic Equations

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Name ______________________________ Period __________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

1. Jebediah and Kalani were working on solving a problem in their Intermediate Algebra class. The original problem was to find the solutions for the following equation.

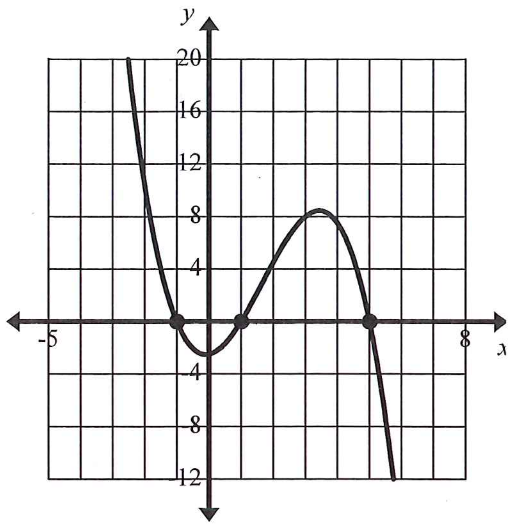
$$-0.5x^3 + 2.5x^2 + 0.5x + 5.5 = 8$$

Jebediah thought that Kalani was doing the problem wrong but they got the same answer.

<u>Jebediah's Method</u>

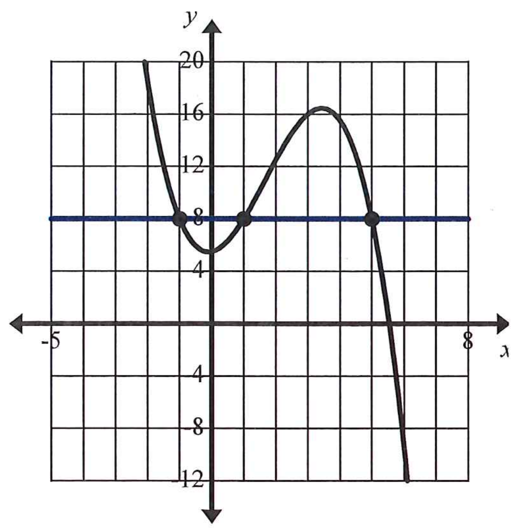
Jebediah got the equation in the form

$-0.5x^3 + 2.5x^2 + 0.5x - 2.5 = 0$ and graphed it. He then looked at the x-intercepts and said that the solutions were $x = -1$, 1 and 5.



<u>Kalani 's Method</u>

Kalani graphed $y = -0.5x^3 + 2.5x^2 + 0.5x + 5.5$ and $y = 8$. Then she found the intersection of the two graphs. She said the solutions were $x = -1$, 1 and 5.



a) Are **both methods** valid? Explain. Yes!

Jebediah knows that setting a function = 0, he can find the solutions (x-intercepts), where the y value is 0.

Kalani put the 2 sides of the equation into her graphing calculator as 2 separate functions and looked at where they were the same. what x values

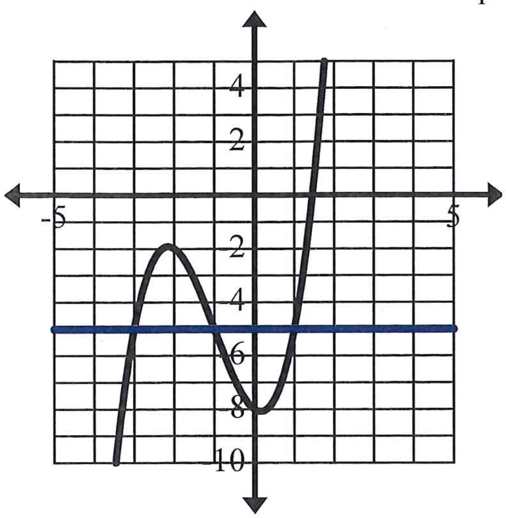
b) Will both methods always work? Why?

Name ______________________________ Period __________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

#2 – 4: Find the solution for each problem. Verify that each answer truly is a solution.

2. $x^3 + 3x^2 - x - 8 = -5$ Proposed Solution(s): $x = -3, -1, 1$



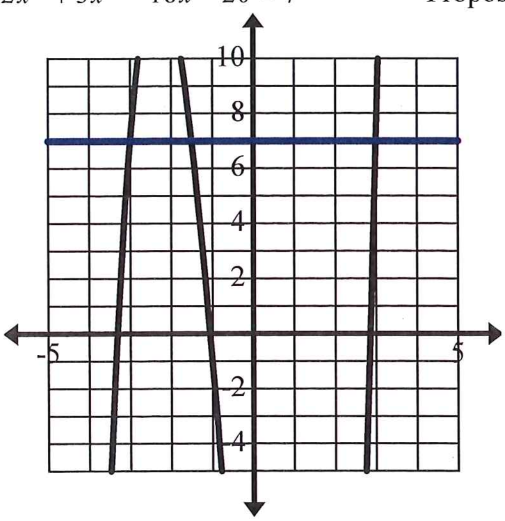
✓ Verify your solution(s):

$(-3)^3 + 3(-3)^2 - (-3) - 8 =$
$-27 + 3(9) + 3 - 8 =$
$-27 + 27 + 3 - 8$
$-5 = -5$ ✓

$(-1)^3 + 3(-1)^2 - (-1) - 8$
$-1 + 3(1) + 1 - 8$
$3 - 8 = -5$ ✓

$(1)^3 + 3(1)^2 - (1) - 8$
$1 + 3 - 1 - 8 = -5$ ✓

3. $2x^3 + 3x^2 - 18x - 20 = 7$ Proposed Solution(s): $x = -3, -1.5, 3$



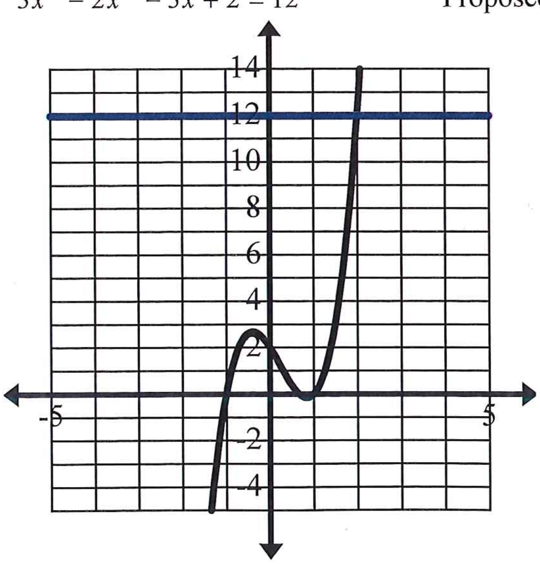
✓ Verify your solution(s):

$2(-3)^3 + 3(-3)^2 - 18(-3) - 20 =$
$2(-27) + 3(9) + 54 - 20$
$-54 + 27 + 54 - 20 = 7$ ✓

$2(-1.5)^3 + 3(-1.5)^2 - 18(-1.5) - 20 =$
$-6.75 + 6.75 + 27 - 20 = 7$
$7 = 7$ ✓

$2(3)^3 + 3(3)^2 - 18(3) - 20 =$
$54 + 27 - 54 - 20 =$
$7 = 7$ ✓

4. $3x^3 - 2x^2 - 3x + 2 = 12$ Proposed Solution(s): $x = 2$



✓ Verify your solution(s):

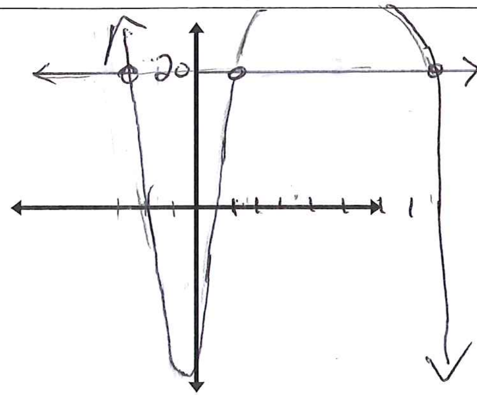
$3(2)^3 - 2(2)^2 - 3(2) + 2 =$
$3(8) - 2(4) - 6 + 2$
$24 - 8 - 6 + 2$
$26 - 14 = 12$ ✓

Name ______________________________ Period __________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

5. Use a graphing utility to identify the solution(s) to each equation. Include a sketch of the graph that appeared. You will need to make adjustments to the window to see all the solutions.

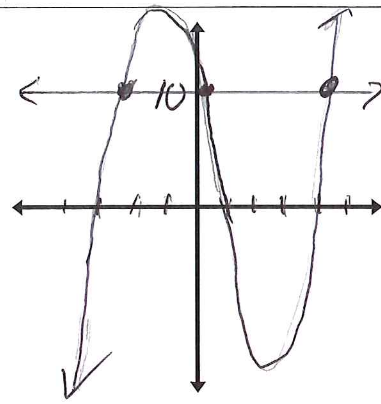
a) $-x^3 + 6.5x^2 + 13x - 8 = 20$



Solutions

x = -2.607,
1.392, or
7.715

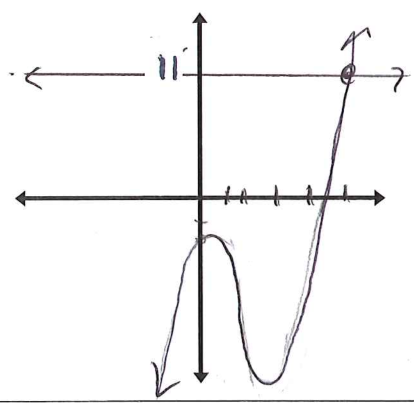
b) $x^3 - 2x^2 - 11x + 12 = 10$



Solutions

x = -2.574,
0.177,
4.398

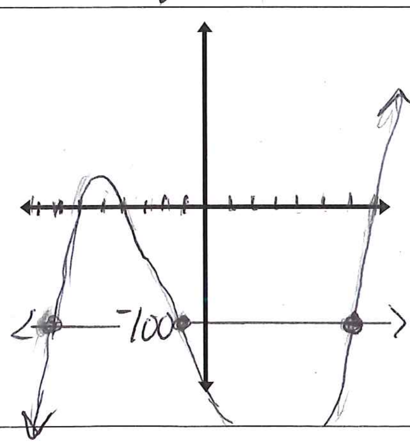
c) $x^3 - 5x^2 + 3x - 2 = 11$



Solutions

x = 4.927

d) $(x+4)(x-7)(x+6) = -100$



Solutions

x = -7.786,
-1.410,
6.196

Name ______________________ Period __________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

6. Find the solution(s) to each equation by graphing.

a) $x^4 - x^3 + 6.5x^2 + 13x - 8 = 20$

$x = -2.067$ or 1.284

b) $x^5 - x^4 + x^3 - 2x^2 - 11x + 12 = 10$

$x = -1.446,$
$0.177,$ or
2.086

c) $\frac{1}{2}x^4 + x^3 - 5x^2 + 3x - 2 = -2$

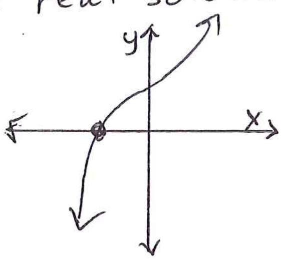
$x = -4.511, 0, 0.759, 1.753$

d) $(x+1)(x+4)(x-7)(x+6) = 25$

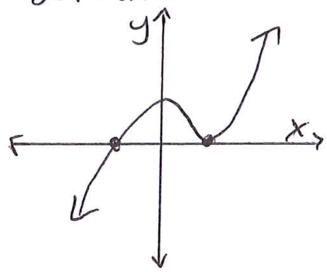
$x = -6.169, -3.623, -1.230, 7.022$

7. Considering the general shape of a cubic function, how many solutions can a cubic equation have? Explain your answer clearly and give an example of each.

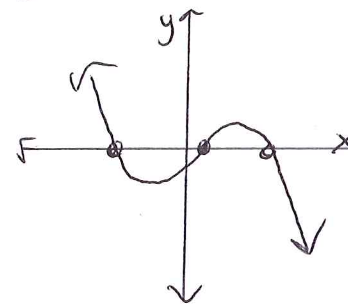
1 real solution



2 real solutions

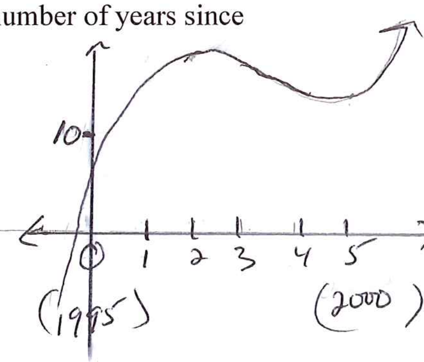


3 real solutions



8. The average amount of bananas (in pounds) eaten per person each year in the United States from 1995 to 2000 can be modeled by $f(x) = 0.298x^3 - 2.73x^2 + 7.05x + 8.45$ where x is the number of years since 1995.

a) Graph the function using a graphing calculator and sketch the graph.



b) In what year did the average number of pounds first reach 14?

In the year 1996 (1.7 years after 1995)

c) When was the average number of pounds equal to 13.5? Explain your thinking.

At 3 different times, the maximum # of solutions for a cubic.

when $x = 1.2$ ⇒ In the year 1996
when $x = 2.7$ ⇒ In the year 1997
when $x = 5.3$ ⇒ In the year 2000

Name ______________________________ Period ____________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

9. Kali invested $2500 for four years in a savings account for 3 years. Her investment is modeled by $s(r) = 2500(1+r)^3$, where r is the annual interest rate written as a decimal.

a) What interest rate will she need if the value of her investment is to grow to $3000? Explain your thinking. 6.3% This is where the line y = 3000 intersects the given model, which is an exponential growth function.

b) What interest rate will she need if the value of her investment is ~~to~~ grow to $5000? Explain your thinking. 26% Using the graph, this is the x-value where the curve intersects the y = 5000 line.

c) What interest rate will she need if the value of her investment is to grow to $8,000? Explain your thinking. 47% This interest rate is unrealistic, so Kali is dreaming if she wants her \$2500 to grow to \$8000 in such a short time!
This is the x-value where the curve intersects the y = 8000 line.

10. Kerry was researching Juvenile crime rates. He found that the yearly number of arrests for crime per 100,000 juveniles from 10 to 17 years of age could be modeled by the function $f(x) = -0.357x^3 + 9.417x^2 - 51.852x + 361.208$, where x is the number of years after 1990.

a) What was the number of arrests in 1990? What point is this on the graph?

361 arrests The y intercept

b) The annual number of arrests peaked at about 501. What year did this happen in?

14.2 years after, so in the year 2004

c) When is the annual number of arrests projected to be 235?

In the year 2010

d) When is it projected that there will be no juvenile arrests? What point on the graph is this?

About 22 years after 1990, so in the year 2012. The x-intercept

Name ______________________ Period __________

6.3B *Finding Real Solutions of Polynomial Equations Graphically*

11. The size of the U.S. workforce (in millions) can be modeled by $W(x) = -0.0001x^3 + 0.0088x^2 + 1.43x + 57.9$, where x is the number of years after 1970.

a) When will the projected workforce be 160 million? Is there more than one period in time where this is expected to happen? In the year 2034 and again in the year 2,108

b) What does the model project the maximum workforce will be? Explain your answer.
189.3 million; this is a relative maximum for time after 1970.

c) When does it predict this will happen? Explain your answer.
when $x = 104.3$ years after 1970, which is in the year 2074.

12. A construction company is building new homes. The median cost of building these homes can be modeled by the function $C(x) = 0.6199x^4 - 55.9808x^3 + 1518.304x^2 - 8252.987x + 30170.846$, where x is the number of years since 1970. In what year was their cost at \$120,000? 1989

13. The graphs of $y = x^3 + x^2 - 5x$ and $y = -x^2 - 4x + 2$ are shown below.

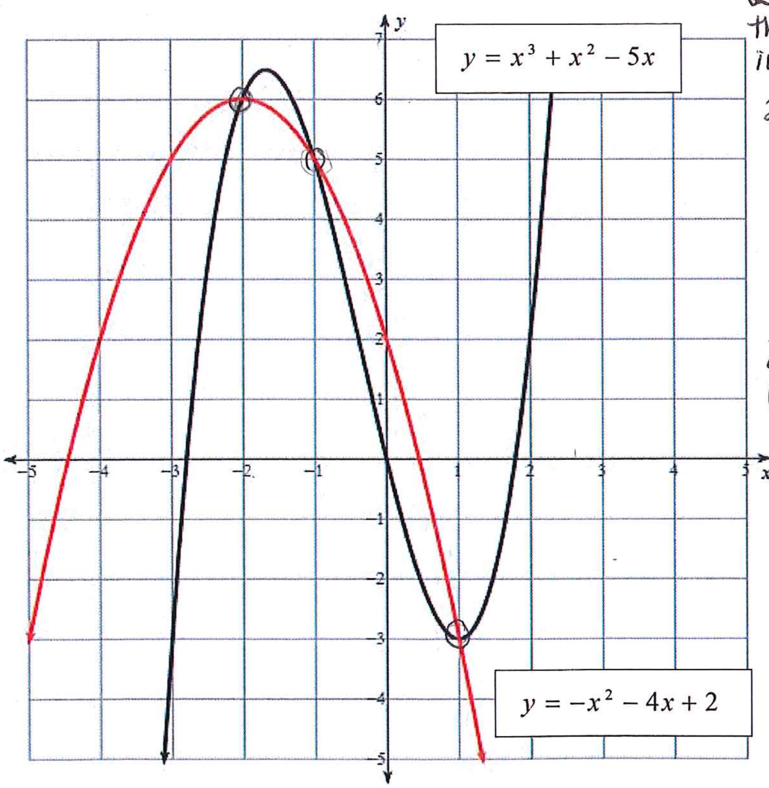
a) Find all points of intersection of the two graphs. $(-2, 6), (-1, 5), (1, -3)$

b) Explain 2 methods for finding the solutions to this problem.

1) using the graphing method as shown here, the solutions are the x-coordinates of the point(s) of intersection, so $\{x = -2, -1, 1\}$

2) Solving algebraically, when does $y_1 = y_2$?

$$x^3 + x^2 - 5x = -x^2 - 4x + 2$$
$$+x^2 + 4x - 2 \qquad +x^2 + 4x - 2$$
$$x^3 + 2x^2 - x - 2 = 0$$
$$x^2(x+2) - 1(x+2) = 0$$
$$(x^2 - 1)(x+2) = 0$$
$$(x+1)(x-1)(x+2) = 0$$
$$\{x = -1, 1, -2\}$$



Section 6.3B

Name ____________________ Period __________

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

1. Find all the zeros of the polynomial function. At least one of the zeros has been provided. (Note - some of the zeros are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). NOTE - all answers MUST be exact – no approximate answers!

a) $y = 2x^3 + 14x^2 + 19x - 2$

$x = -2$

$-2 \,|\, 2 \quad 14 \quad 19 \quad -2$

$\quad -4 \quad -20 \quad 2$

$2 \quad 10 \quad -1 \,|\, 0$

$2x^2 + 10x - 1 = 0$

$x = \dfrac{-10 \pm \sqrt{(10)^2 - 4(2)(-1)}}{2(2)}$

$= \dfrac{-10 \pm \sqrt{100+8}}{4} = \dfrac{-10 \pm \sqrt{108}}{4}$

$= \dfrac{-10 \pm 6\sqrt{3}}{4} = \dfrac{-5 \pm 3\sqrt{3}}{2}$

$x = -2, \dfrac{-5 \pm 3\sqrt{3}}{2}$

b) $f(x) = x^3 - x^2 - 12x + 90$

$x = -5$

$-5 \,|\, 1 \quad -1 \quad -12 \quad 90$

$\quad -5 \quad 30 \quad -90$

$1 \quad -6 \quad 18 \,|\, 0$

$x^2 - 6x + 18 = 0$

$x^2 - 6x + 9 = -18 + 9$

$\sqrt{(x-3)^2} = \sqrt{-9}$

$|x-3| = 3i$

$x = 3 \pm 3i$

$x = -5, 3 \pm 3i$

c) $y = x^4 - 3x^3 - 20x^2 + 50x$

$x = 0, x = 5$

$y = x(x^3 - 3x^2 - 20x + 50)$

$5 \,|\, 1 \quad -3 \quad -20 \quad 50$

$\quad 5 \quad 10 \quad -50$

$1 \quad 2 \quad -10 \,|\, 0$

$x^2 + 2x - 10 = 0$

$x^2 + 2x + 1 = 10 + 1$

$\sqrt{(x+1)^2} = \sqrt{11}$

$|x+1| = \sqrt{11}$

$x = -1 \pm \sqrt{11}$

$x = 0, 5, -1 \pm \sqrt{11}$

2. Find all the solutions of the polynomial equation. At least one of the factors has been provided. (Note - some of the solutions are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). NOTE - all answers MUST be exact - no approximate answers!

a) $x^3 - 7x^2 - 22x + 160 = 0$

$(x-5)$

$5 \,|\, 1 \quad -7 \quad -22 \quad 160$

$\quad 5 \quad -10 \quad -160$

$1 \quad -2 \quad -32 \,|\, 0$

$x^2 - 2x - 32 = 0$

$x^2 - 2x + 1 = 32 + 1$

$\sqrt{(x-1)^2} = \sqrt{33}$

$|x-1| = \sqrt{33}$

$x = 1 \pm \sqrt{33}$

$x = 5, 1 \pm \sqrt{33}$

b) $3x^3 + 19x^2 - 3x - 3 = 0$

$(3x+1)$

$-\frac{1}{3} \,|\, 3 \quad 19 \quad -3 \quad -3$

$\quad -1 \quad -6 \quad 3$

$3 \quad 18 \quad -9 \,|\, 0$

$\dfrac{3x^2 + 18x - 9}{3} = \dfrac{0}{3}$

$x^2 + 6x - 3 = 0$

$x^2 + 6x + 9 = 3 + 9$

$\sqrt{(x+3)^2} = \sqrt{12}$

$|x+3| = 2\sqrt{3}$

$x = -3 \pm 2\sqrt{3}$

$x = -\frac{1}{3}, -3 \pm 2\sqrt{3}$

c) $x^4 + 9x^3 + 36x^2 + 54x = 0$

x and $(x+3)$

$x(x^3 + 9x^2 + 36x + 54) = 0$

$x = 0, \quad -3 \,|\, 1 \quad 9 \quad 36 \quad 54$

$\quad -3 \quad -18 \quad -54$

$1 \quad 6 \quad 18 \quad 0$

$x^2 + 6x + 18 = 0$

$x^2 + 6x + 9 = -18 + 9$

$\sqrt{(x+3)^2} = \sqrt{-9}$

$|x+3| = 3i$

$x = -3 \pm 3i$

$x = 0, -3, -3 \pm 3i$

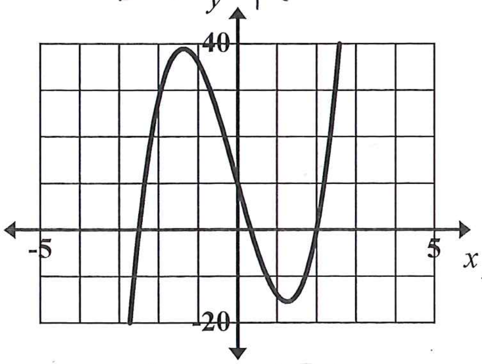
Name ______________________ Period ________

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

3. Find all the x-intercepts of the polynomial function. Give exact answers. (Note - some of the x-intercepts are irrational numbers and cannot be expressed as fractions or decimals). The graph of the function is provided. You may use your graphing calculator to obtain a better graph if you like. NOTE - all answers MUST be exact - no approximate answers!

a) $y = 6x^3 + x^2 - 31x + 10$

$$\begin{array}{r|rrrr} 2 & 6 & 1 & -31 & 10 \\ & & 12 & 26 & -10 \\ \hline & 6 & 13 & -5 & 0 \end{array}$$



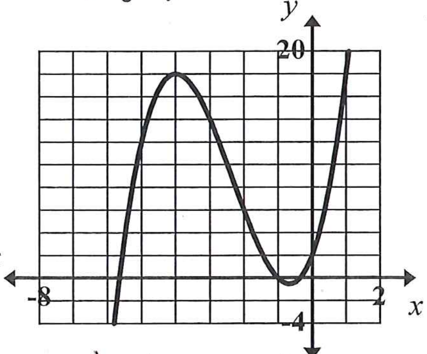
$6x^2 + 13x - 5 = 0$

$(3x-1)(2x+5) = 0$

$\boxed{x = 2, \frac{1}{3}, -\frac{5}{2}}$

b) $y = x^3 + 7x^2 + 8x + 2$

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 8 & 2 \\ & & -1 & -6 & -2 \\ \hline & 1 & 6 & 2 & 0 \end{array}$$



$x^2 + 6x + 2 = 0$

$x^2 + 6x + 9 = -2 + 9$

$\sqrt{(x+3)^2} = \sqrt{7}$

$|x+3| = \sqrt{7}$

$\boxed{x = -1, -3 \pm \sqrt{7}}$

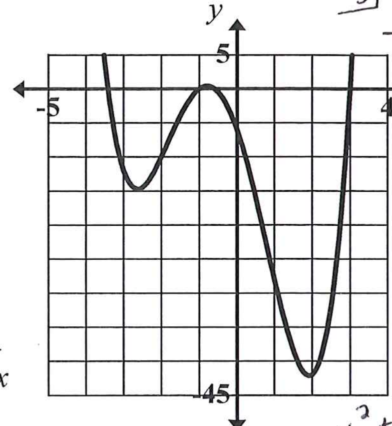
c) $y = x^4 + 2x^3 - 9x^2 - 16x - 6$

$$\begin{array}{r|rrrrr} 3 & 1 & 2 & -9 & -16 & -6 \\ & & 3 & 15 & 18 & 6 \\ \hline & 1 & 5 & 6 & 2 & 0 \end{array}$$

$x^3 + 5x^2 + 6x + 2$

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 6 & 2 \\ & & -1 & -4 & -2 \\ \hline & 1 & 4 & 2 & 0 \end{array}$$

$x^2 + 4x + 2 = 0$



$x^2 + 4x + 4 = -2 + 4$

$\sqrt{(x+2)^2} = \sqrt{2}$

$|x+2| = \sqrt{2}$

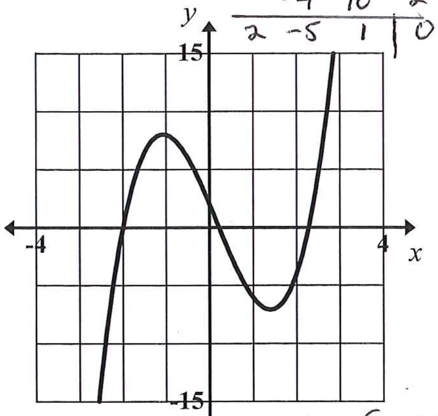
$x = -2 \pm \sqrt{2}$

$\boxed{x = 3, -1, -2 \pm \sqrt{2}}$

4. Find all the zeros of the polynomial function. Give exact answers. (Note - some of the zeros are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). The graph of the function is provided. . You may use your graphing calculator to obtain a better graph if you like. NOTE - all answers MUST be exact - no decimals!

a) $f(x) = 2x^3 - x^2 - 9x + 2$

$$\begin{array}{r|rrrr} -2 & 2 & -1 & -9 & 2 \\ & & -4 & 10 & -2 \\ \hline & 2 & -5 & 1 & 0 \end{array}$$



$2x^2 - 5x + 1 = 0$ $a = 2, b = -5, c = 1$

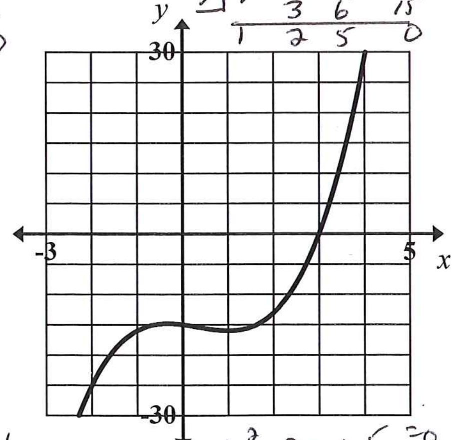
$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$

$= \dfrac{5 \pm \sqrt{25 - 8}}{4}$

$\boxed{x = -2, \frac{5 \pm \sqrt{17}}{4}}$

b) $g(x) = x^3 - x^2 - x - 15$

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -1 & -15 \\ & & 3 & 6 & 15 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$



$x^2 + 2x + 5 = 0$

$x^2 + 2x + 1 = -5 + 1$

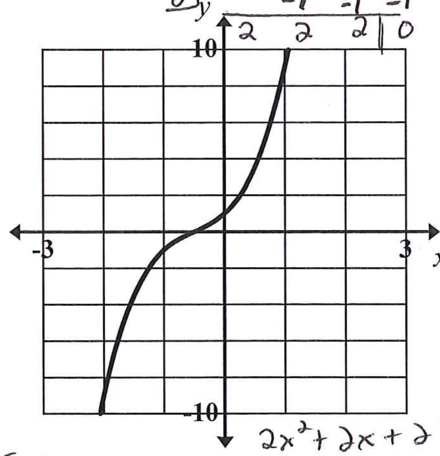
$\sqrt{(x+1)^2} = \sqrt{-4}$

$|x+1| = 2i$

$\boxed{x = 3, -1 \pm 2i}$

c) $h(x) = 2x^3 + 3x^2 + 3x + 1$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & 3 & 1 \\ & & -1 & -1 & -1 \\ \hline & 2 & 2 & 2 & 0 \end{array}$$



$2x^2 + 2x + 2 = 0$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{-3}}{2}$

$\boxed{x = -\frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{2}}$

Name ______________________________ Period __________

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

5. Find all the solutions to the polynomial equation $f(x)=0$. Give exact answers. (Note - some of the solutions are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph).

a) $f(x)=x^3+x^2-5x-2$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -5 & -2 \\ & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & 0 \end{array}$$

$x^2+3x+1=0$

$x=\frac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$

$x=\frac{-3\pm\sqrt{5}}{2}$

$\boxed{x=2,\ \frac{-3\pm\sqrt{5}}{2}}$

b) $f(x)=x^4-4x^3+4x^2-64$

X-ints at -2 and 4

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & 4 & 0 & -64 \\ & & -2 & 12 & -32 & 64 \\ \hline & 1 & -6 & 16 & -32 & 0 \end{array}$$

$x^3-6x^2+16x-32=0$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 16 & -32 \\ & & 4 & -8 & 32 \\ \hline & 1 & -2 & 8 & 0 \end{array}$$

$x^2-2x+8=0$

$x^2-2x+1=-8+1$

$\sqrt{(x-1)^2}=\sqrt{-7}$

$|x-1|=i\sqrt{7}$

$x=1\pm i\sqrt{7}$

$\boxed{x=-2,\ 4,\ 1\pm i\sqrt{7}}$

c) $f(x)=x^4-2x^3-14x^2+30x+9$

$*$ Double zero at $x=3$

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & -14 & 30 & 9 \\ & & 3 & 3 & -33 & -9 \\ \hline & 1 & 1 & -11 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -11 & -3 \\ & & 3 & 12 & 3 \\ \hline & 1 & 4 & 1 & 0 \end{array}$$

$x^2+4x+1=0$

$x^2+4x+4=-1+4$

$\sqrt{(x+2)^2}=\sqrt{3}$

$|x+2|=\sqrt{3}$

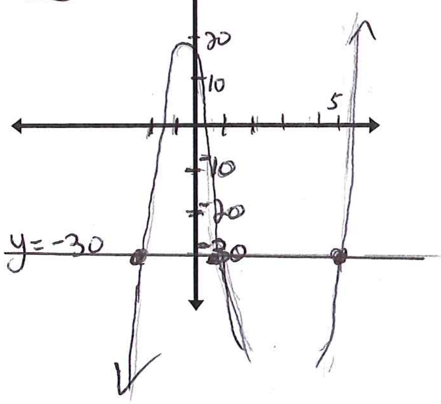
$x=-2\pm\sqrt{3}$

$\boxed{x=3^*,\ -2\pm\sqrt{3}}$

6. Use a graphing utility to identify the solution(s) to each equation. Include a sketch of the graph that appeared. You will need to make adjustments to the window to see all the solutions. Find ALL solutions (real and imaginary) to these equations. All solutions must be exact – no approximate answers.

a) $4x^3-15x^2-31x=-30$

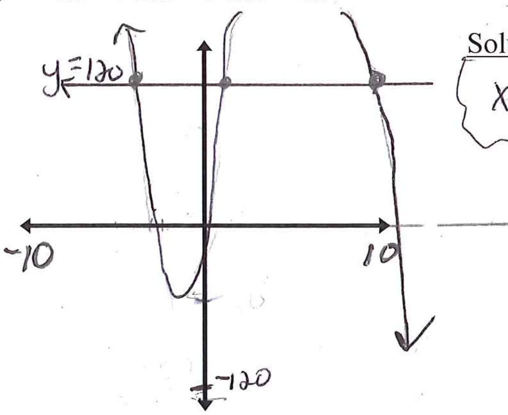
$x=5,\ -2,\ 0.75$ ← Solutions



b) $-2x^3+15x^2+62x=120$

Solutions

$x=-4,\ 1.5,\ 10$

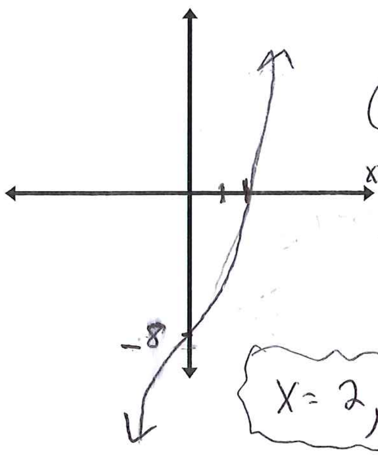


6.3C *Finding All Solutions of Polynomial Equations Algebraically*

6. (continued) Use a graphing utility to identify the solution(s) to each equation. Include a sketch of the graph that appeared. You will need to make adjustments to the window to see all the solutions. Find ALL solutions (real and imaginary) to these equations. All solutions must be exact – no approximate answers.

c) $2x^3 - 16 = 0$

$2(x^3 - 8) = 0$



Solutions

$(x)^3 - (2)^3 = 0$

$(x-2)(x^2+2x+4) = 0$

$x = 2$, $x^2+2x+1 = -4+1$

$\sqrt{(x+1)^2} = \sqrt{-3}$

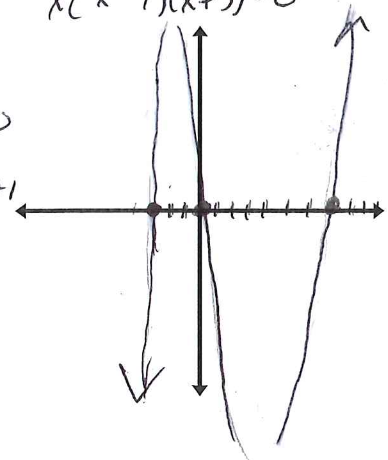
$|x+1| = i\sqrt{3}$

$x = -1 \pm i\sqrt{3}$

$X = 2, -1 \pm i\sqrt{3}$

d) $x\left(x^2 - 4x - 21\right) = 0$

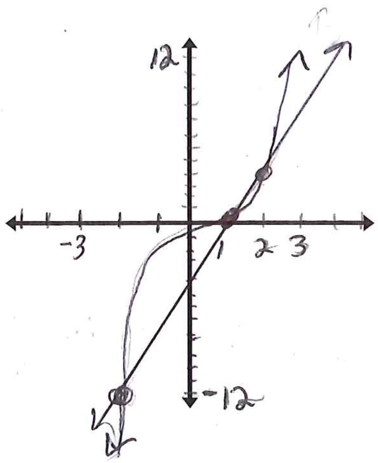
$x(x-7)(x+3) = 0$



Solutions

$x = -3, 0, 7$

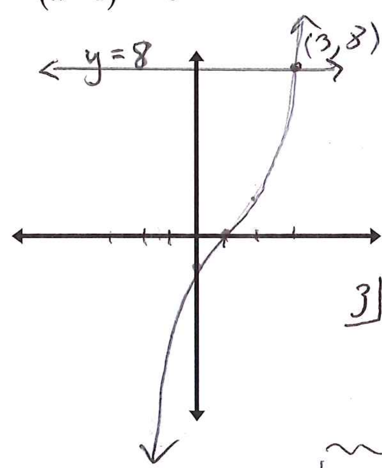
e) $x^3 - x^2 = 4x - 4$



Solutions

$x = -2, 1, 2$

f) $(x-1)^3 = 8$



Solutions

$(x-1)(x-1)(x-1) = 8$

$(x-1)(x^2-2x+1)$

$x^3 - 3x^2 + 3x - 1 - 8 = 0$

$x^3 - 3x^2 + 3x - 9 = 0$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 3 & -9 \\ & & 3 & 0 & 9 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{-3}$

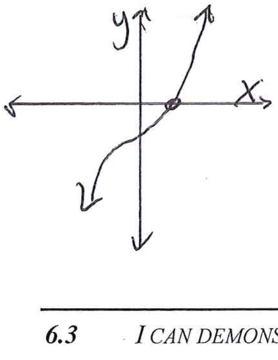
$x = \pm i\sqrt{3}$

$X = 3, \pm i\sqrt{3}$

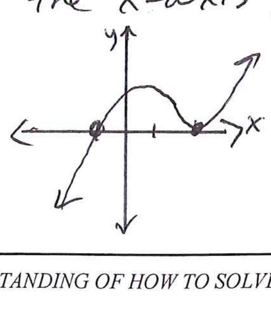
7. How many solutions can a cubic function have? Explain your answer clearly and give an example of each.

A cubic function always has 3 solutions. Either:

1 real & 2 imaginary solutions,

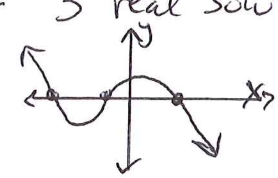


2 different real solutions But one of them is a duplicate Solution where the graph just touches the x-axis, like



$x = -1, 2, 2$ but written $x = -1, 2$

or 3 real solutions



Section 6.3C

Name ______________________________ Period __________

Unit 6 *Review Materials*

1. Considering the equation $f(x) = x^3 - x^2 - 4x + 4$

a) What form is this polynomial function written in?

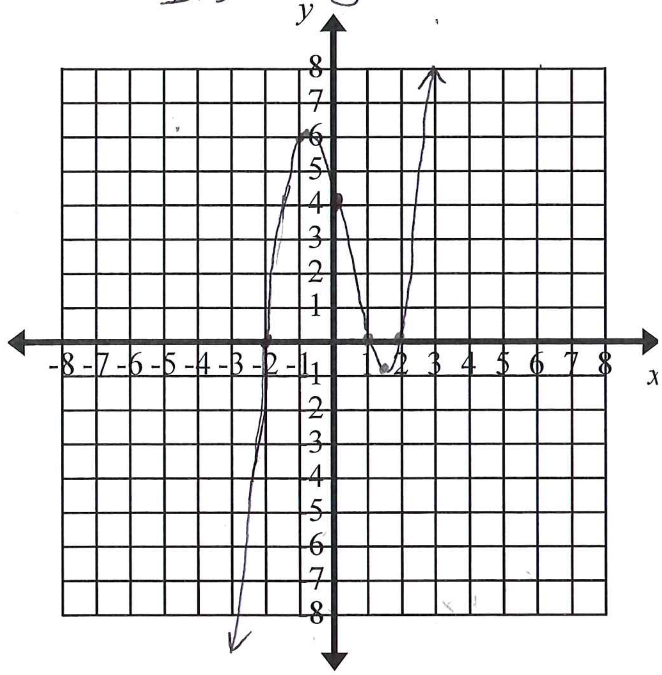
Standard form

b) What is the significance of the "+ 4"?

It's the y-intercept

b) Use a graphing utility to create a table of values and graph the function.

x	y
-3	-20
-2	0
-1	6
0	4
1	0
2	0
3	10



c) Find the information listed below:

Relative minimum: (1.54, -0.88)

Relative maximum: (-0.87, 6.06)

Domain: all real numbers

Range: all real numbers

increasing interval(s): $x < -0.87$, $x > 1.54$

decreasing interval(s): $-0.87 < x < 1.54$

zero(s): (-2, 0), (1, 0), (2, 0)

2. Convert each to standard form.

a) $y = -3(x-2)^2 + 5$

$= -3(x^2 - 4x + 4) + 5$

$= -3x^2 + 12x - 12 + 5$

$\boxed{y = -3x^2 + 12x - 7}$

b) $y = (x+5)(x-7)(x+4)$

$(x+5)(x^2 - 3x - 28)$

$x^3 - 3x^2 - 28x$

$+ 5x^2 - 15x - 140$

$\boxed{y = x^3 + 2x^2 - 43x - 140}$

3. Considering the equation $f(x) = -x(x-1)(x+2)$

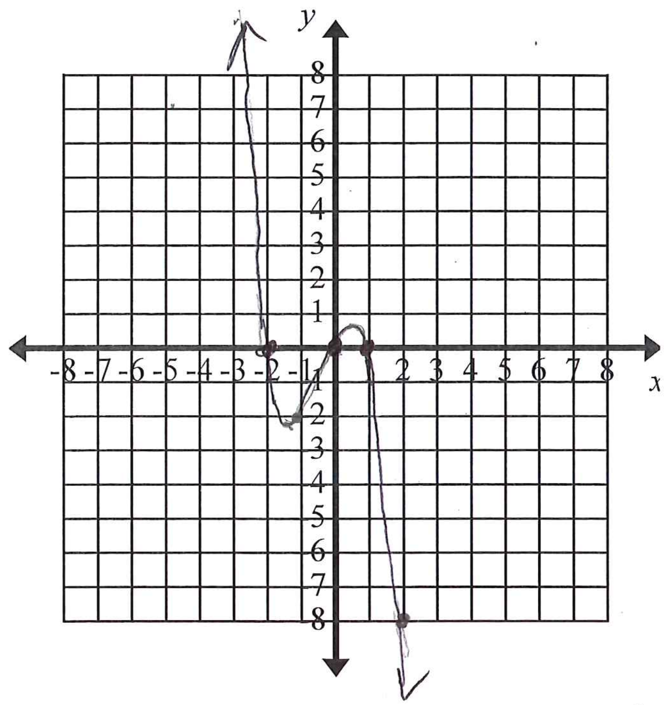
a) What form is this polynomial function written in?

Intercept form or factored form

b) What is the obvious significant feature given in this form? X-intercept(s)

c) Use a graphing utility to create a table of values and graph the function.

x	y
-3	12
-2	0
-1	-2
0	0
1	0
2	-8
3	-30



d) Find the information listed below:

Relative minimum: (-1.22, -2.11)

Relative maximum: (0.55, 0.63)

Domain: all real numbers

Range: all real numbers

increasing interval(s): $-1.22 < x < 0.55$

decreasing interval(s): $x < -1.22$, $x > 0.55$

zero(s): (0, 0), (1, 0), (-2, 0)

y-intercept: (0, 0)

Name ______________________ Period __________

Unit 6 *Review Materials*

4. Considering the equation $f(x) = 0.5(x-1)^3 + 4$

a) What form is this polynomial function written in?

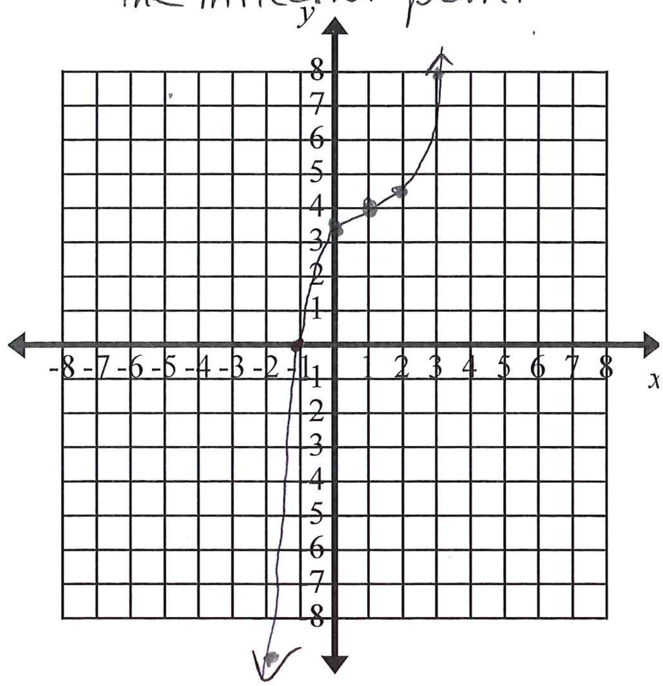
It is similar to the vertex form of a quadratic equation, but this is a cubic equation,

b) What is the obvious significant feature given in this form?

the inflection point

c) Use a graphing utility to create a table of values and graph the function.

x	y
-2	-9.5
-1	0
0	3.5
1	4
2	4.5
3	8



d) Find the information listed below:

Relative minimum: none

Relative maximum: none

Domain: all real numbers

Range: all real numbers

increasing interval(s): $-\infty < x < \infty$

decreasing interval(s): none

x-intercept(s): $(-1, 0)$

y-intercept: $(0, 3.5)$

5. Simplify using the indicate operation.

a) $(2x^3 + 5x^2 - 5) + 2(6x^3 - 2x) - (3x^2 - 7x + 1)$

$2x^3 + 5x^2 - 5 + 12x^3 - 4x - 3x^2 + 7x - 1$

$\boxed{14x^3 + 2x^2 + 3x - 6}$

b) $(5x^3 - 2x^2 + 7) - (8x^2 - 11)$

$5x^3 - 2x^2 + 7 - 8x^2 + 11$

$\boxed{5x^3 - 10x^2 + 18}$

c) $(x+1)(2x+3)$

$\boxed{2x^2 + 5x + 3}$

d) $(3x-5)(x+1)(x+6)$

$(3x-5)(x^2+7x+6) =$

$3x^3 + 21x^2 + 18x$

$- 5x^2 - 35x - 30$

$\boxed{3x^3 + 16x^2 - 17x - 30}$

5. (continued) Simplify using the indicateD operation.

e) $2(x+4)^3-5$

$2\left[(x+4)(x+4)(x+4)\right]-5$

$(x+4)(x^2+8x+16)$

x^3+8x^2+16x

$+4x^2+32x+64$

$2(x^3+12x^2+48x+64)-5=$

$\boxed{2x^3+24x^2+96x+123}$

f) $(x^3-10x^2+27x-12)\div(x-4)$

4	1	-10	27	-12
		4	-24	12
	1	-6	3	0

$\boxed{x^2-6x+3}$

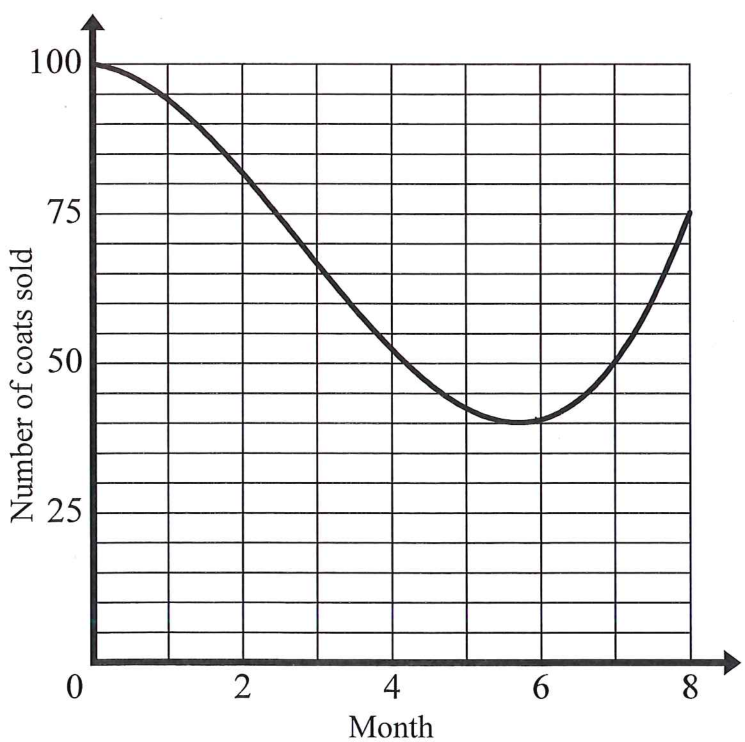
g) $(x^4-3x^3+8x^2-2)\div(x+2)$

-2	1	-3	8	0	-2
		-2	10	-36	72
	1	-5	18	-36	70

$\boxed{x^3-5x^2+18x-36+\frac{70}{x+2}}$

6. The graph below shows the number of coats sold each month at a department store where $t = 1$ represents January of the current year.

a) In what month number did they sell the least amount of coats?
During month #5



b) How many coats did they sell in month zero? 100 Coats

c) How many coats did they sell in month 7? 50 Coats

d) State the interval(s) when coat sales were increasing.

$x > 5.5$

e) State the interval(s) when coat sales were decreasing.

$0 < x < 5.5$

f) Did they ever sell zero coats in a month? No

Name ______________________________ Period __________

Unit 6 *Review Materials*

7. Given one factor of the polynomial, use synthetic division to help you rewrite the polynomial in factored form. Show your work!

a) $f(x) = x^3 - 7x^2 - x + 7$

A factor of this polynomial is $(x - 7)$

$$\begin{array}{r|rrrr} 7 & 1 & -7 & -1 & 7 \\ & & 7 & 0 & -7 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$x^2 - 1$

$\boxed{f(x) = (x+1)(x-1)(x-7)}$

b) $f(x) = x^3 - 2x^2 - 11x + 12$

A factor of this polynomial is $(x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -11 & 12 \\ & & -3 & 15 & -12 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$x^2 - 5x + 4$

$\boxed{f(x) = (x-1)(x-4)(x+3)}$

8. Find the rational zeros of two of the equations **graphically**.

Find the rational zeros of two of the equations **algebraically**.

Use each equation only once.

(a) $y = x^3 + 5x^2 - 9x - 45$

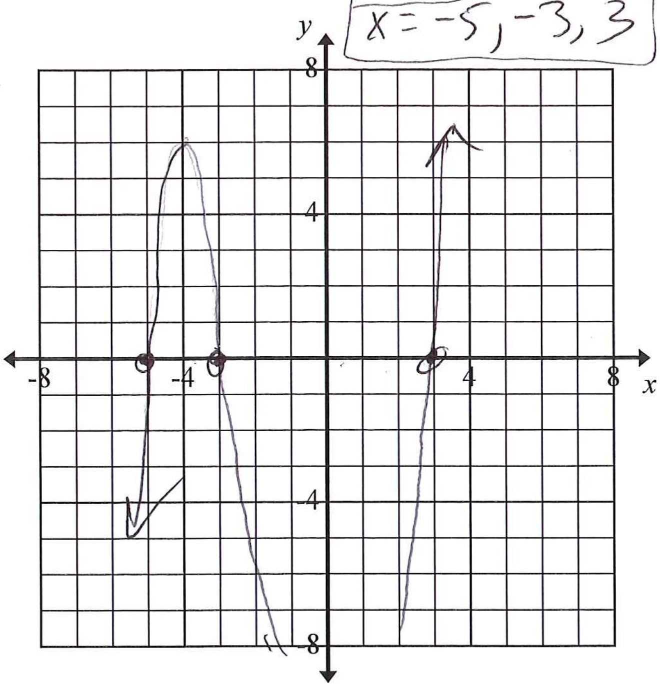
(b) $y = (x+5)(x-2)(3x+4)$

(c) $y = x(2x-1)(3x+1)$

(d) $y = x^3 + 4x^2 + x - 6$

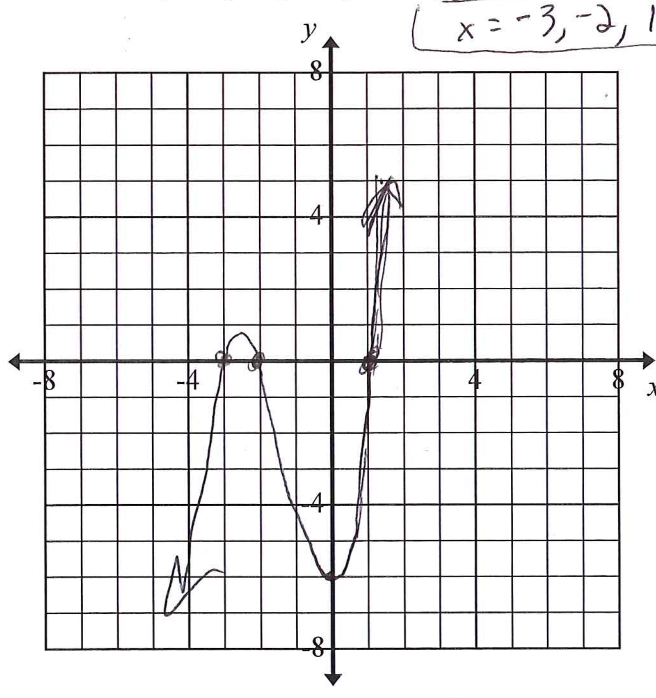
Solve by Graphing: equation (a)

$\boxed{x = -5, -3, 3}$



Solve by Graphing: equation (d)

$\boxed{x = -3, -2, 1}$



Solve algebraically: equation (c)

$x(2x-1)(3x+1) = 0$

$\boxed{x = 0,\ x = \tfrac{1}{2},\ x = -\tfrac{1}{3}}$

Solve algebraically: equation (b)

$(x+5)(x-2)(3x+4) = 0$

$\boxed{x = -5, 2, -\tfrac{4}{3}}$

Name ____________________ Period __________

Unit 6 *Review Materials*

9. Given one zero (x-intercept) of the polynomial, use synthetic division to find the other zeros. Show your work! (Do not graph.)

a) $f(x) = 2x^3 - 3x^2 - 23x + 12$

A zero of this polynomial is -3

$-3 \,|\; 2 \quad -3 \quad -23 \quad 12$

$\quad\quad\;\; -6 \quad 27 \quad -12$

$\quad 2 \quad -9 \quad 4 \quad 0$

$2x^2 - 9x + 4 = 0$

$(2x-1)(x-4) = 0$

$\boxed{x = 1/2,\ 4,\ \text{or } -3}$

b) $f(x) = 2x^3 + 7x^2 - 28x + 12$

An x-intercept of this polynomial is 2

$2 \,|\; 2 \quad 7 \quad -28 \quad 12$

$\quad\quad\;\; 4 \quad 22 \quad -12$

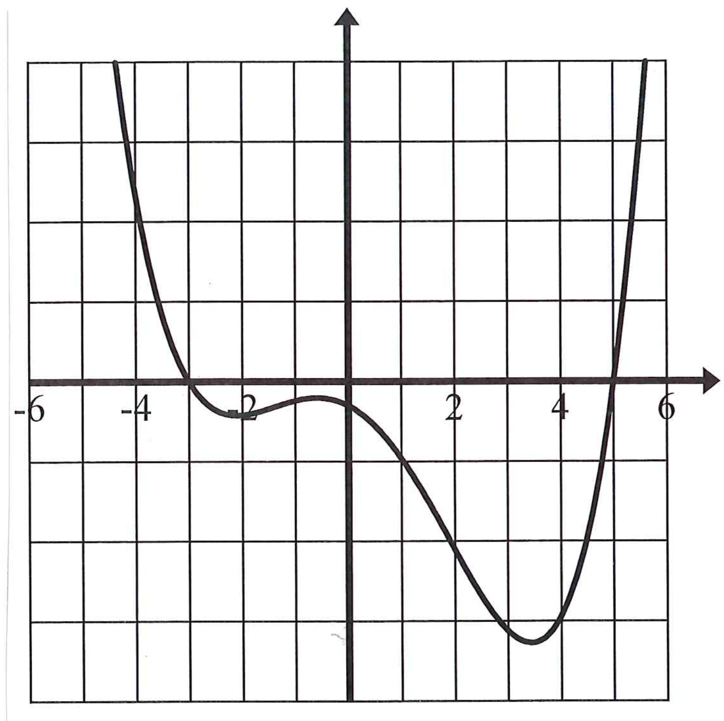
$\quad 2 \quad 11 \quad -6 \quad 0$

$2x^2 + 11x - 6 = 0$

$(2x-1)(x+6) = 0$

$\boxed{x = \frac{1}{2},\ -6,\ \text{or } 2}$

10. Using the graph to the right identify the rational roots. Use these, and the function below to aid in finding additional roots that exist for this 4th degree polynomial. List all 4 roots.



$\boxed{f(x) = x^4 - x^3 - 16x^2 - 17x - 15}$

$5 \,|\; 1 \quad -1 \quad -16 \quad -17 \quad -15$

$\quad\quad\quad 5 \quad 20 \quad 20 \quad 15$

$-3 \,|\; 1 \quad 4 \quad 4 \quad 3 \;\|\; 0$

$\quad\quad\; -3 \quad -3 \quad -3$

$\quad 1 \quad 1 \quad 1 \;\|\; 0$

$(x^2 + x + 1) = 0$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

$x = \frac{-1 \pm i\sqrt{3}}{2}$

$x = 5$
$x = -3$
$x = \frac{-1 \pm i\sqrt{3}}{2}$

11. Use your graphing calculator to help you get started. Find all zeros of each function below. Give answers as exact, no approximations.

a) $f(x) = x^4 + 4x^3 + 8x^2 + 11x + 6$

$-2 \,|\; 1 \quad 4 \quad 8 \quad 11 \quad 6$

$\quad\quad\;\; -2 \quad -4 \quad -8 \quad -6$

$\quad 1 \quad 2 \quad 4 \quad 3 \quad 0$

$-1 \,|\; 1 \quad 2 \quad 4 \quad 3$

$\quad\quad\;\; -1 \quad -1 \quad -3$

$\quad 1 \quad 1 \quad 3 \quad 0$

$(x^2 + x + 3) = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{-11}}{2}$

$x = \frac{-1 \pm i\sqrt{11}}{2}$

$x = -2$
$x = -1$
$x = \frac{-1 \pm i\sqrt{11}}{2}$

b) $f(x) = x^4 - 10x^3 + 34x^2 - 43x + 12$

$3 \,|\; 1 \quad -10 \quad 34 \quad -43 \quad 12$

$\quad\quad\;\; 3 \quad -21 \quad 39 \quad -12$

$4 \,|\; 1 \quad -7 \quad 13 \quad -4 \;\|\; 0$

$\quad\quad\;\; 4 \quad -12 \quad 4$

$\quad 1 \quad -3 \quad 1 \;\|\; 0$

$(x^2 - 3x + 1) = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{5}}{2}$

$x = 3$
$x = 4$
$x = \frac{3 \pm \sqrt{5}}{2}$

Unit 6 Review